Magnetic field perturbations in the systems where only poloidal magnetic field is present*

D.D. Ryutov (LLNL), J. Kesner (MIT),
M.E. Mauel (Columbia Univ.)

Presented at APS DPP Meeting, Monday, October 27, 2003

+Work performed for the U.S. DOE by UC LLNL under contract W-7405-Eng-48 and by MIT and CU under contract DE-FG02-91ER54109.

Abstract

In some plasma confinement systems the confinement is provided by a poloidal magnetic field (no toroidal magnetic field is present). Examples include FRC, levitated dipoles, and long diffuse pinches. We consider the influence of the magnetic field perturbations on the structure of the magnetic field in such systems and find that the effect of perturbations is quite different from that in the systems where a substantial toroidal field is present. In particular, even infinitesimal perturbations can, in principle, lead to large radial excursions of the field lines. The particle motion in such systems is a competition between the (toroidal) curvature drift across the field lines and the radial excursions due to streaming along the field. Introduction of a weak regular toroidal magnetic field reduces radial excursions of the field lines. Possible source of perturbations could be low-frequency non-MHD modes in plasmas with a large enough beta values, as well as imperfections of the coils.

OUTLINE

- Motivation
- Simple example of a “rectified” system
- Small perturbations in the general geometry
- Uniform magnetic field imposed on the levitated dipole and elongated FRC
- Numerical examples and mitigating effects
- Comments on the neoclassical transport
- Conclusion

Motivation

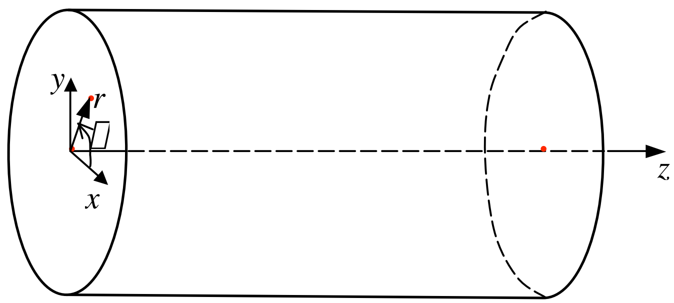
There exist confinement systems where only poloidal magnetic field is present: levitated dipole, FRC, long diffuse pinch (the latter case is equivalent to a “rectified” dipole)

Effect of magnetic perturbations on the magnetic field structure in such systems cannot be described in terms of the familiar island formation (and possible overlapping) as is the case in the systems with a substantial toroidal field, like tokamaks, RFPs, spheromaks.

It turns out that the magnetic field structure in the “poloidal field only” systems is very sensitive to perturbations.

In this paper we concentrate on the magnetic field structure, and only in passing refer to effect of perturbations on transport.

"RECTIFIED" SYSTEM CONTAINS MOST IMPORTANT FEATURES OF A FULL PROBLEM



The unperturbed magnetic field has only θ component, $B_\theta = B_{\theta 0}(r) \equiv B_0(r)$; $B_{r0} = B_{z0} = 0$. θ and z are analogs of the poloidal and toroidal coordinates, respectively

Perturbations:

$\delta B_r(r,\theta,z),\ \delta B_\theta(r,\theta,z),\ \delta B_z(r,\theta,z)$.

Introduce poloidally-averaged perturbations:

$$\delta\bar{B}_{r,\theta,z}(r,z) = \frac{1}{2\pi}\int_0^{2\pi} \delta B_{r,\theta,z}(r,\theta,z)\,d\theta \qquad \delta\mathbf{B}_1 \equiv \delta\mathbf{B} - \delta\bar{\mathbf{B}};\ \int_0^{2\pi}\delta\mathbf{B}_1\,d\theta = 0$$

The equations for the perturbed magnetic field line are:

$$\frac{dr}{d\varphi}=\frac{\delta\bar{B}_r+\delta B_{1r}}{B_0+\delta B_\varphi};\quad \frac{dz}{d\varphi}=\frac{\delta\bar{B}_z+\delta B_{1z}}{B_0+\delta B_\varphi}$$

Only toroidally-averaged perturbations make contributions of the first order to displacement of the field line during one revolution around the axis ($\Delta\varphi=2\pi$).

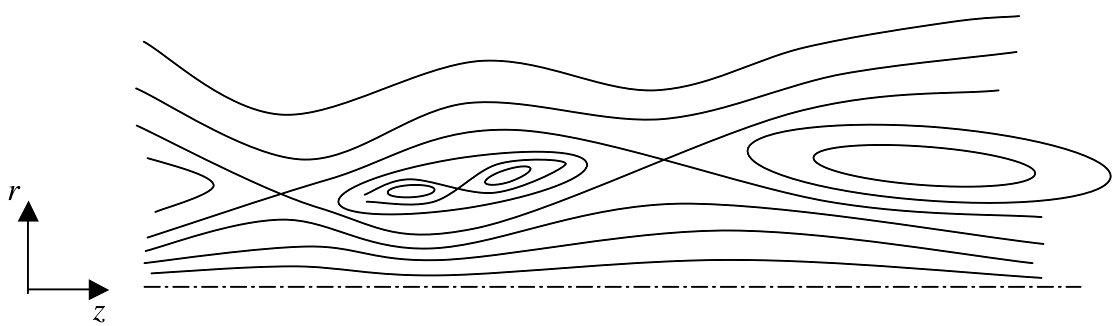
To the first order,

$$\frac{dr}{dz}=\frac{\delta\bar{B}_r(r,z)}{\delta\bar{B}_z(r,z)} \qquad (*)$$

As one has $\delta\bar{B}_r(r,z)=-\frac{\partial A(r,z)}{\partial z};\ \delta\bar{B}_z(r,z)=\frac{1}{r}\frac{\partial}{\partial r}\left[rA(r,z)\right]$, Eq. (*) has an integral

$$rA(r,z)=const$$

This equation describes puncture plots produced by the perturbed field lines in the *(r,z)* plane



A remarkable feature of the "poloidal-field-only" systems: even an infinitesimal perturbation causes a dramatic change of the magnetic topology: without perturbations, each field line would have generated only a point on the (r,z) plane, determined by the initial z and r.

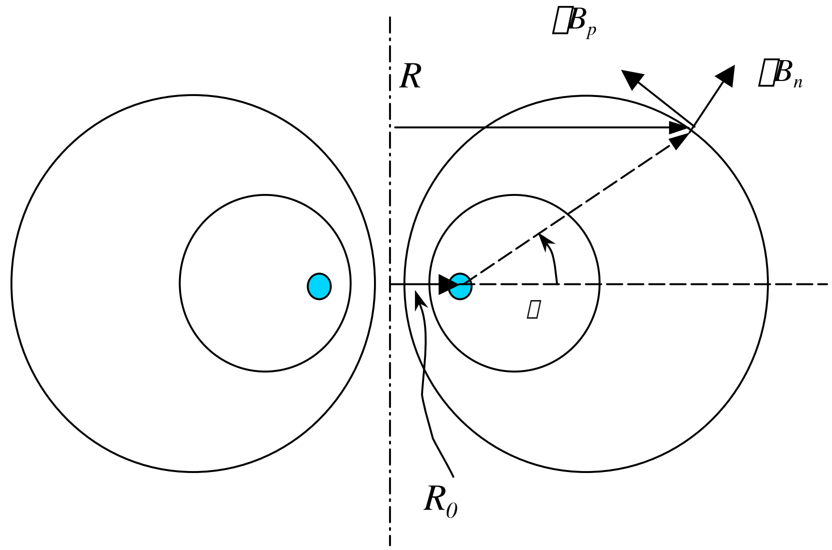
The path s (the number of turns around the axis) that particular field line makes before getting displaced by a substantial distance in the radial direction is, of course, determined by the amplitude of the perturbation. For the field line to be displaced radially by characteristic size a (of order of a radial length-scale of perturbations), the field line has to make a path

$$s \sim a \frac{B_0}{\Delta B} \qquad (**)$$

General geometry: use coordinates ψ (poloidal flux), l (length of the unperturbed field line measured from the equatorial plane), ϕ (toroidal angle).

We decompose magnetic field perurbations over three mutually perpendicular directions: the direction normal to the flux surface ("radial"), the direction tangential to the flux surface and lying in the poloidal plane, and the toroidal directions. We denote these components by δB_n, δB_p, δB_t, respectively.

An example: a magnetic field of a current ring



In the linear approximation,

$$\frac{d\square}{dl} = 2\square R\square B_n; \quad \frac{d\square}{dl} = \frac{\square B_t}{B_0 R}$$

The (small) variation of $\square$ and $\square$ during one full turn in the poloidal direction is:

$$\square\square = 2\square\oint R\square B_n dl; \quad \square\square = \oint\frac{\square B_t dl}{RB_0} \qquad (***)$$

Right-hand sides of these equations are functions of $\square$ and $\square$.

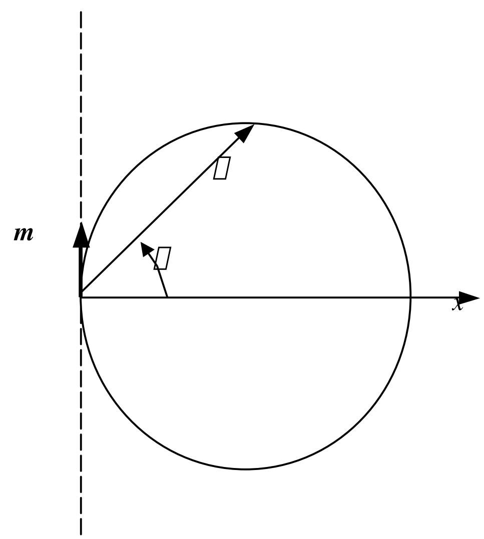
$$\frac{d\square}{d\square} = \frac{2\square\oint R\square B_n dl}{\oint\frac{\square B_t dl}{RB_0}} \equiv F(\square,\square)$$

This result is quite general in that it is not based on any assumptions about the plasma beta. It is equally applicable to the levitated dipoles, FRCs, and diffuse pinches.

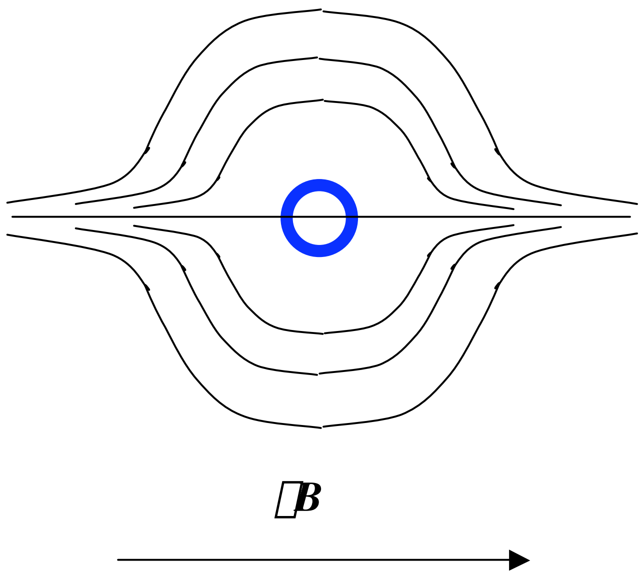
An illustration: perturbation in the form of a uniform magnetic field perpendicular to the axis of the device and directed along the axis x:

$$\Box B_n = b n_x \cos \Box ; \quad \Box B_t = \Box b \sin \Box$$

In the case of a point dipole



A puncture plot for the equatorial plane



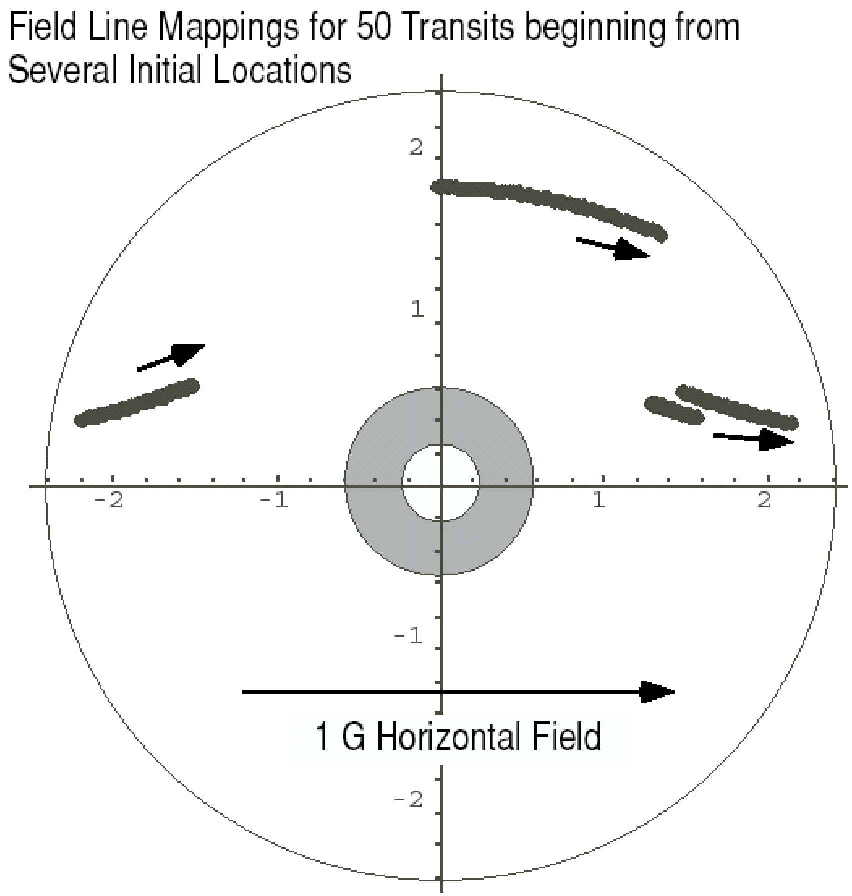
$$\frac{1}{R}\frac{dR}{d\theta} = -\alpha \cot\theta \; ; \; \alpha \approx 0.7$$

The family of puncture plots is

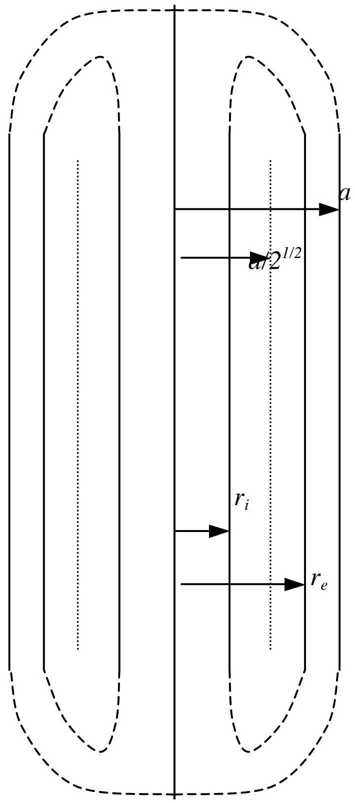
$$R = Const\left|\sin\theta\right|^{-\alpha}$$

Shown in blue, is the ring

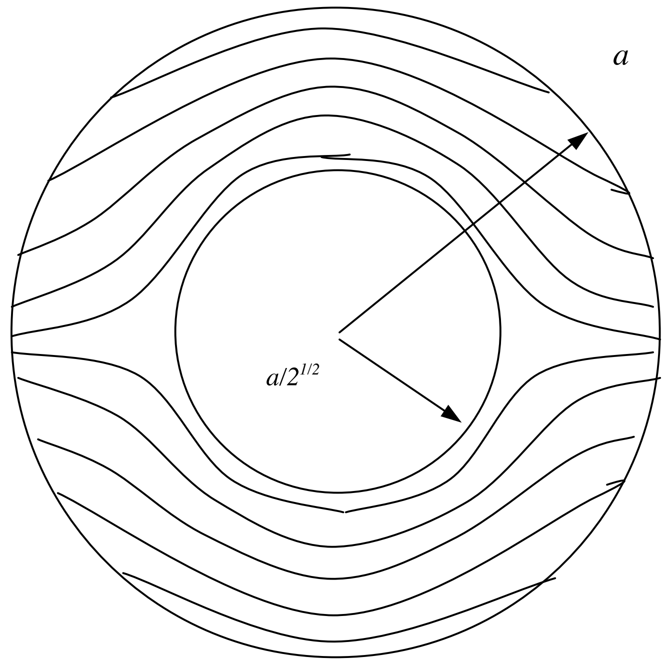
Direct integration of the field line equation



A racetrack-shaped FRC ($L>>a$)



Puncture plots in the equatorial plane



Are the effects discussed in this paper significant?

A numerical example: a system with the minor radius a~2 m designed for a confinement of a plasma with electron temperature T_e~30 keV, exposed to a perturbation with a relative amplitude

$$\varepsilon \sim \delta B / B_0 \sim 10^{-3}$$

At this amplitude, according to Eq. (**), the length of the field line before it gets displaced radially by the distance ~a is ~2000 m. For the 30-keV electron, the transit time over such a distance is a mere 20 μs, many orders of magnitude less than the required confinement time (for typical MFE densities of ~ 10^{14} cm^{-3}).

Plasma transport is determined by particle trajectories, which may strongly deviate from magnetic field lines.

We briefly discuss these effects for the case of a levitated dipole

Particles with a sufficiently large pitch angle experience mirror reflection from the zone of a strong magnetic field inside the ring. These particles are bouncing along a segment of a field line between the turning points. They will not cover the whole field line and execute large radial excursions. Conversely, transit particles would cover the whole field line and would suffer from large radial excursions.

Consider now effects of particle drift. To be specific, we refer our estimates to a levitated dipole. If the toroidal drift velocity is large, so that toroidal displacement caused by this drift within one poloidal period of the particle motion exceeds toroidal displacement [illegible] [Eq. ***] caused by the presence of a perturbing magnetic field, radial excursions of particles decrease compared to the global scale of the system.

The criterion for this to happen reads as

$$\frac{r_C}{R} > \frac{b}{B_0} \qquad (****)$$

where r_C is the cyclotron radius of a particle. Clearly, this condition is most restrictive for the electrons. Taking R=200 cm, T_e~30 keV, and the magnetic field B_0~1 T, on finds that the relative value of perturbations must be small, $b/B_0 < 3 \cdot 10^{-4}$.

Radial excursions of transiting particles under condition (****) will be, by the order of magnitude,

$$\delta r \sim R\frac{Rb}{r_C B_0}$$

Assuming that the magnetic field in the center of the ring is 10 T, one can evaluate the number of transiting particles as ε~1/10. Assuming that the time for one radial excursion is shorter than the electron scattering time over the loss-cone angle, $\tau \sim \varepsilon/\nu_e$, one finds the corresponding neoclassical diffusion coefficient:

$$D \sim \varepsilon\frac{\delta r^2}{\tau} \sim \nu_e R^2\left(\frac{Rb}{r_C B_0}\right)^2$$

The corresponding electron diffusion time across the plasma volume would be $\sim (1/\nu_e)\left(r_C B_0 / Rb\right)^2$. If the inequality (****) holds by a 10-fold margin, i.e., the relative perturbation level is ~$3\cdot10^{-5}$, the confinement time turns out to be ~100 electron scattering times, i.e., quite short.

The presence of a radial electric field complicates the situation. Depending on its sign, the EXB drift is directed oppositely to the gradB drift for at least one of the particle species. As the EXB drift does not depend on particle energy, there would always exist a group of particles in the velocity space for which these two drifts cancel each other (we mean here toroidal drift averaged over a period of the poloidal motion). Quantitative evaluation of this effect requires some effort.

One more factor has to be taken into account for the case where perturbations are created by a non-steady-state convection. If the characteristic correlation time is shorter than the time within which an electron makes a full radial excursion, the fluctuating nature of perturbations becomes important. This factor may both decrease the transport and increase it (the latter would occur for the electron whose drift velocity resonates with phase velocity of perturbations).

For mitigating these effects, one can add a weak toroidal magnetic field

In order for this field to have a significant effect, it must satisfy the condition

$$B_{t0} >> \delta B.$$

At the same time, we do not want to make the system to become a more “traditional” confinement system of the type of a spheromak or RFP where the toroidal and poloidal field are of the same order of magnitude. So, we assume that

$$B_{t0} << B_0.$$

The latter inequality shows that, to the first order, one can use Eq. (***) for tracing the field lines, with the only difference being that now one has to use B_{t0} instead of δB_t. This yields:

$$\frac{d\psi}{d\phi} = \frac{2\pi \oint R \delta B_n dl}{\oint \frac{B_{t0} dl}{R B_0}}$$

In this case the radial wandering of the field line becomes smaller, of the order of

$$a \delta B / B_{to}$$

Summary

- We developed a general technique for considering magnetic field perturbations in the systems where the unperturbed field has only the poloidal component (levitated dipoles, FRC's, long diffuse pinches)

- Large radial excursions of magnetic field lines may occur even at very small perturbations

- The effect can be mitigated by introducing small toroidal magnetic field

- Particle drifts and particle collisions lead to neoclassical diffusion which, in some cases, can be large