NIMROD Simulations of the Levitated Dipole Experiment

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ABSTRACT

Theory predicts that the closed field line geometry of a levitated dipole configuration can support MHD stable plasmas at high beta values when the pressure gradient does not exceed a critical value. For sufficiently strong heating this stability limit would be violated and we expect instability to develop, leading non-linearly to the formation of convective cells. The NIMROD code [1] permits the study of the non-linear development of MHD modes, including resistive effects. NIMROD simulations have been performed for the LDX geometry. The simulations indicate the onset of MHD activity for sufficient heating power and for realistic (non ideal) heating profiles. In the presence of instability the plasma is seen to exhibit large scale convective flows but it remains centered in the vacuum chamber, anchored by the internal ring. For sufficient heating power we observe that we can exceed the ideal pressure limit in the presence of large-scale convection.

[1] C.R. Sovinec and the Nimrod team, Journal of Computational Physics, Vol. 195, p. 355 (2004).

MHD Equilibrium and Stability of a Dipole

- MHD Equilibrium: Equilibrium is obtained for all β. At high β plasma expands in midplane.

  Free boundary equilibrium: Ref: Garnier, Kesner, Mauel, Phys Plasmas 6 (1999) 3431.

  Analytic equilibrium for point dipole: Krasheninnikov, Catto, Hazeltine, PRL **82** (1999) 2689.

- Ideal Stability:

  - Interchange stability when $\delta(pV^{\gamma}) > 0$ with $V = \oint d\ell/B$.

    Early Refs: Rosenbluth & Longmire, Ann Phys. **1** (1957) 120.

    Bernstein, Frieman, Kruskal, Kulsrud, Proc. R. Soc. London, Ser. A, **244** (1958) 17.

  - Ballooning modes

    Ballooning modes stable when interchange modes stable: Garnier et al, Ibid.

- Resistive MHD: Can have weak resistive mode $\gamma \propto \eta$, but the $\gamma \propto \eta^{1/3}$ mode is not present.

  Ref: Simakov, Catto, Ramos, Hastie, Phy Pl **9** (2002) 4985.

- Non-Linear MHD in hard-core pinch geometry (high aspect ratio approx):

  - Convective cells will develop when interchange limit is exceeded. *Leads to particles transport but not necessarily energy transport.*

    Ref: Reduced MHD: Pastukhov and Chudin, Plasma Physics Report, **27**, (2001) 963.

    PIC simulation: Tonge, Leboeuf, Huang, Dawson, Phys Pl **10** (2003) 3475.

- **We want to simulate MHD instability in non-linear regime in real 3-D dipole geometry**

    Consider configuration similar to LDX with strong localized heating so as to drive instability

NIMROD Code

- Solves non-linear resistive MHD equations
- Initial value calculation in real geometry

  Provide realistic heating and particle source

  Approximate $\chi_\perp = constant$. Use higher order (p=3) elements to permit $\chi_\parallel \gg \chi_\perp$.

  Kinematic viscosity approximation in momentum equation. No diamagnetic terms.

- NIMROD equations

$$\frac{\partial \mathbf{B}}{\partial \mathbf{t}} = -\nabla \times \mathbf{E} + \kappa_{divb} \nabla \nabla \cdot \mathbf{B}$$

$$\mathbf{E} = -\mathbf{U} \times \mathbf{B} + \eta \mathbf{J}$$

$$\mu_0 \mathbf{J} = \nabla \times \mathbf{B}$$

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{U}) = \nabla \cdot D \nabla n$$

$$m_i n (\frac{\partial \mathbf{U}}{\partial t} + \mathbf{U} \cdot \nabla \mathbf{U}) = \mathbf{J} \times \mathbf{B} - \nabla p + m_i \nabla \cdot \nu n \nabla \mathbf{U}$$

$$\frac{n}{\gamma - 1}\left(\frac{\partial T}{\partial t} + \mathbf{U}\cdot\nabla T\right) = -\frac{p}{2}\nabla\cdot\mathbf{U} - \nabla\cdot\mathbf{q} + \mathbf{Q}$$

$\mathbf{U}$ = flow velocity, Q the heat source density, $\mathbf{q}$ the heat flux:

$$\mathbf{q} = -n\left[\chi_{\|}\hat{\mathbf{b}}\hat{\mathbf{b}} + \chi_{\perp}(\mathbf{I} - \hat{\mathbf{b}}\hat{\mathbf{b}})\right]\cdot\nabla T$$

Boundary Conditions: Dirichlet for $\mathbf{B}_{\perp}$, T, $\mathbf{U_i}$. Flux for n, ($F_n = -D\nabla n$).

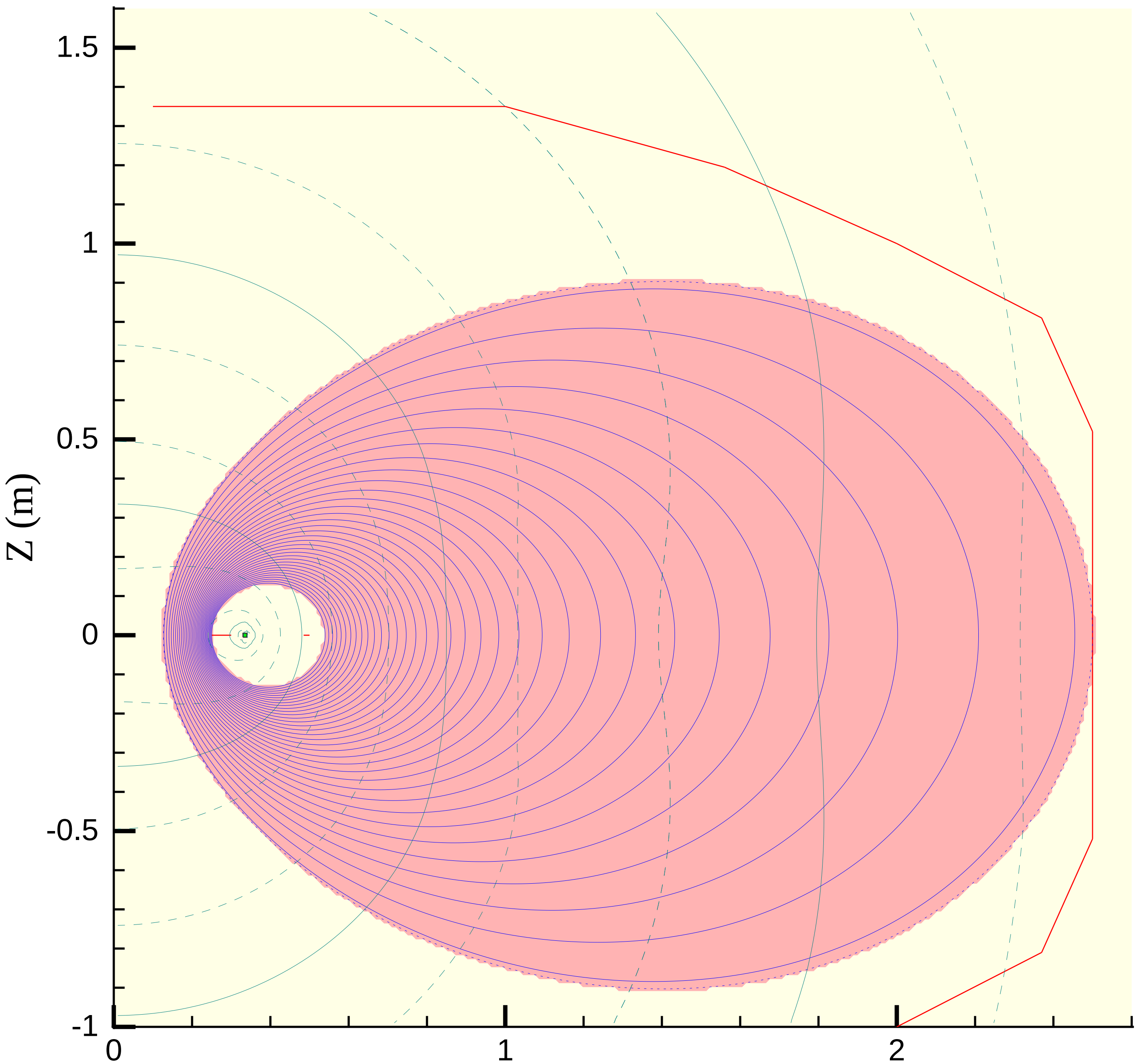
In ideal case dipole can heat up to maximum pressure determined by $pV^\gamma = constant$ profile.

- DIPEQ code produces Grad-Shafranov equilibrium for $pV^\gamma = constant$. For $\beta_{max} = 36\%$ equilibrium for LDX paramters are:

Dipole (LDX) Parameters

Floating coil major radius	0.35 m
Coil Current	1.5×10^6 A
Vacuum vessel midplane radius	2.5 m
Peak field near coil	5 T
Edge pressure	1.3 Pa
Peak pressure	150 Pa
Midplane radius of $R(p_{max})$	1.16 m
Peak β	0.36

Psi and |B| Contours



Nimrod Input Plasma Parameters

Edge Temperature		0.4 eV
Perp thermal diffusivity ($\chi_\perp$)		1 m^2/s
Parallel thermal diffusivity ($\chi_\parallel$)		10^5 m^2/s
Resistivity/μ_0		0.1 - 10 m^2/s
Kinematic viscosity		10 - 1000 m^2/s
Alfven growth rate		0.1-1.5$\times 10^6$ s^{-1}
Heating function **:	Power	120 KW
	$R_0(Q_{Max})$	1.15 m
	FWHM	0.32 m

Other Nimrod Input Parameters

Poloidal grid	30 x 60
Number of toroidal modes	42
Finite element basis function degree	3 †

** High power so as to 1) drive instability & 2) Speed up calculation

† High order finite element necessary to eliminate $\chi_\parallel$ cross-field heat leakage in high β system

Stages of Discharge

- Stable heating: 75 μs (depends on heating rate)

  Pressure increases without observable losses

- Linear instability growth: 125 μs

  Higher-n modes grow fastest

  Pressure increases without observable losses

- Quasi non-linear stage: 200 μs

  Saturation of modes at macroscopic levels

  Pressure profile broadens to approach $pV^\gamma = constant$

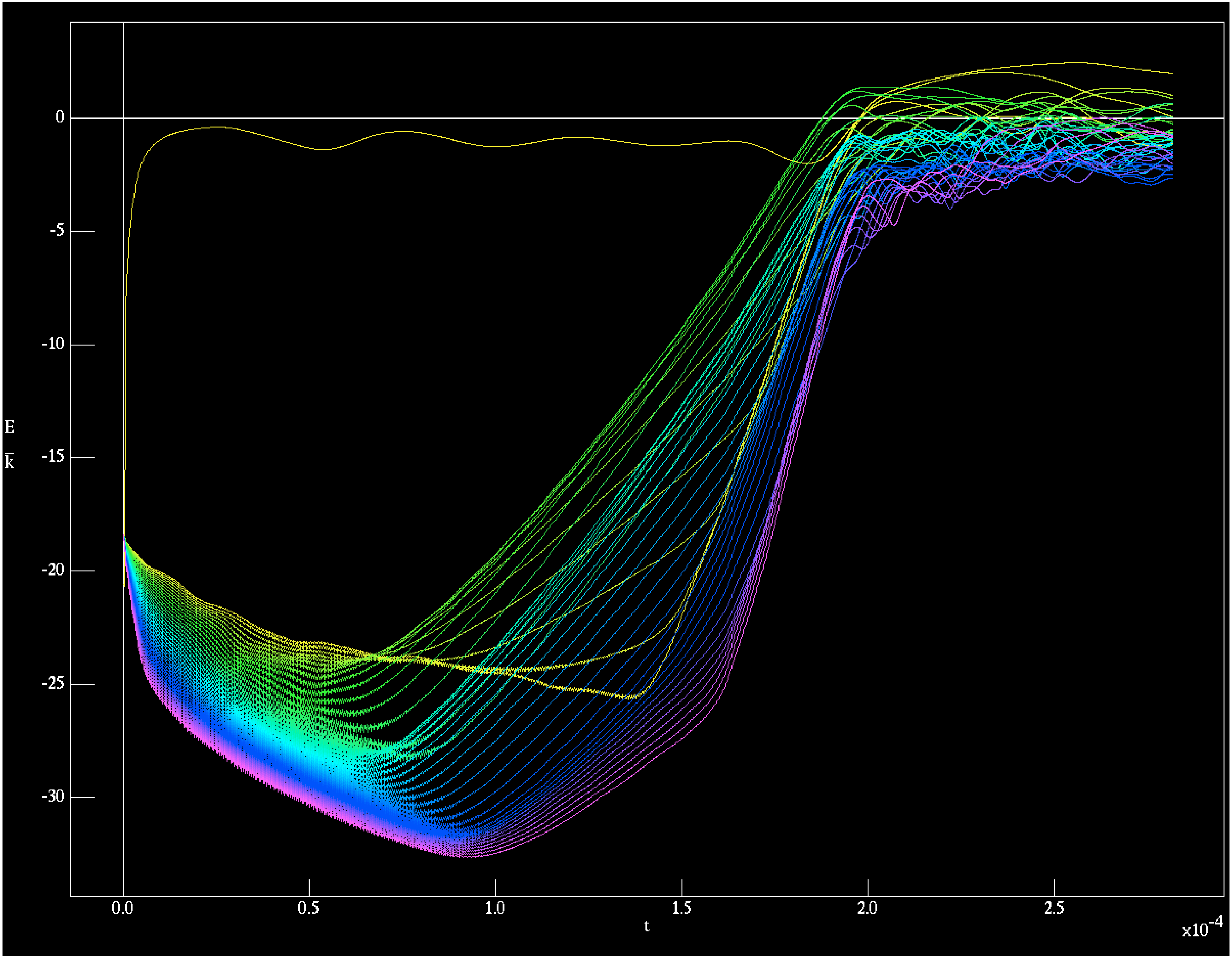
- Saturated state including convection

  convective cells present → n=1 is dominant

  Not steady state but convection pattern persists

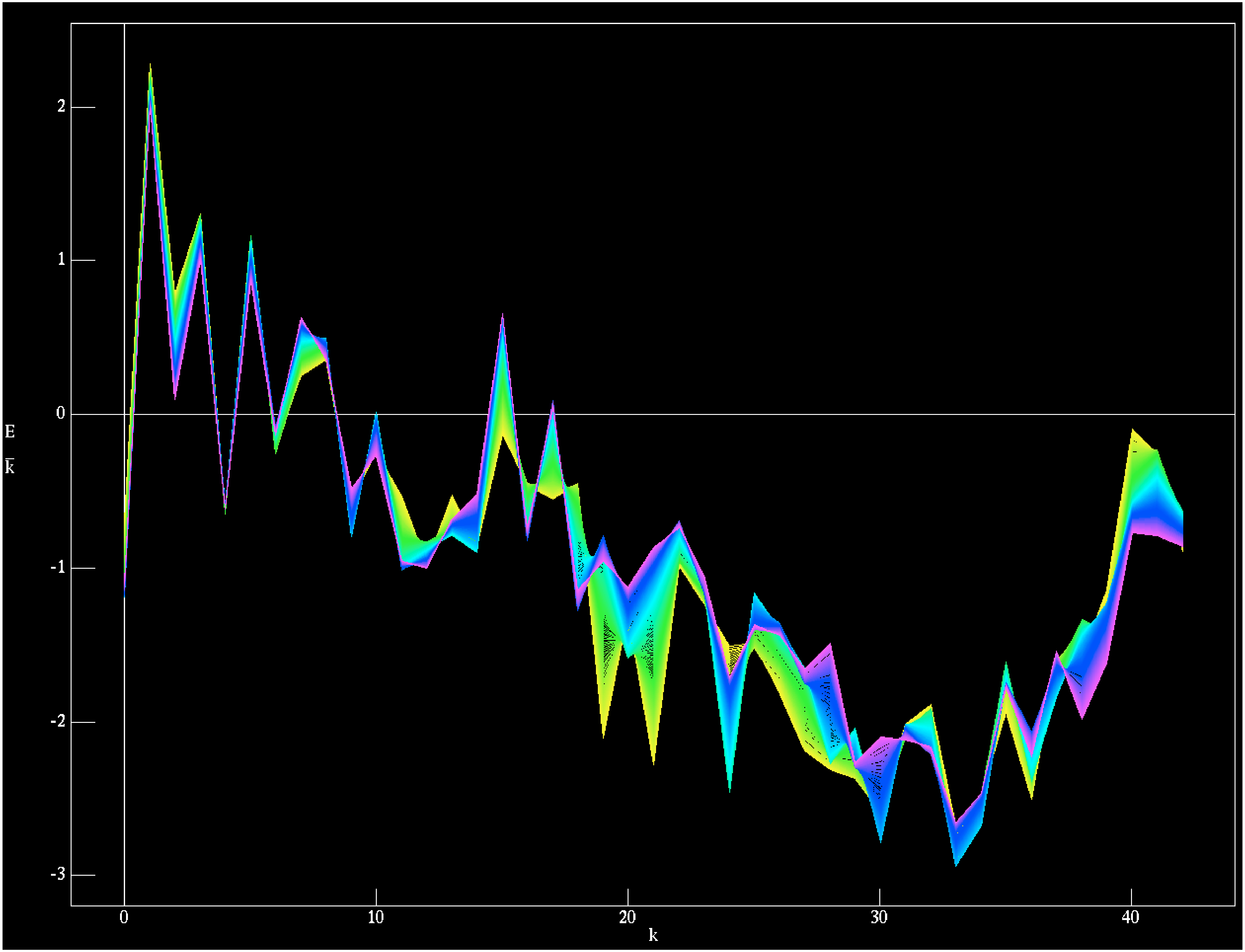
  Similar behavior seen by Pastukhov in Z-pinch geometry

Growth of modes between $0 < t < 0.28\ ms$

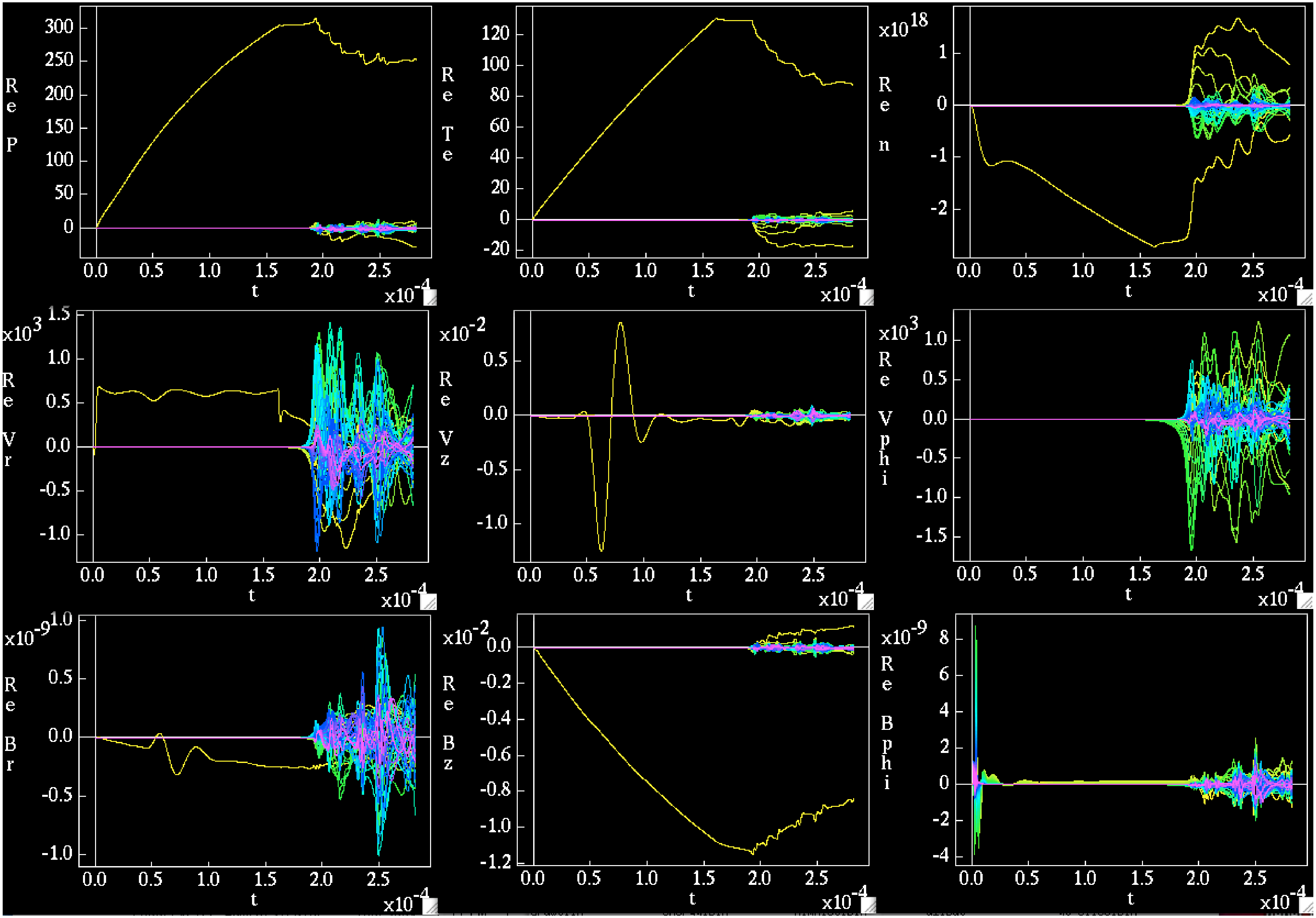

0
-5
-10
-15
-20
-25
-30
E
k̄
0.0
0.5
1.0
1.5
2.0
2.5
t
x10^-4

- $0 < t < 0.075\ ms$: p(ψ) rises linearly w/o instability
- $0.075 < t < 0.18\ ms$: p(ψ) rises linearly w/o instability
- $0.18 < t < 0.5\ ms$: quasi-non-linear regime. pressure profile spreads out to fill vacuum chamber.
- $0.5 < t < 1\ ms$: non-linear regime. pressure profile maintains form in presence of convective cells.

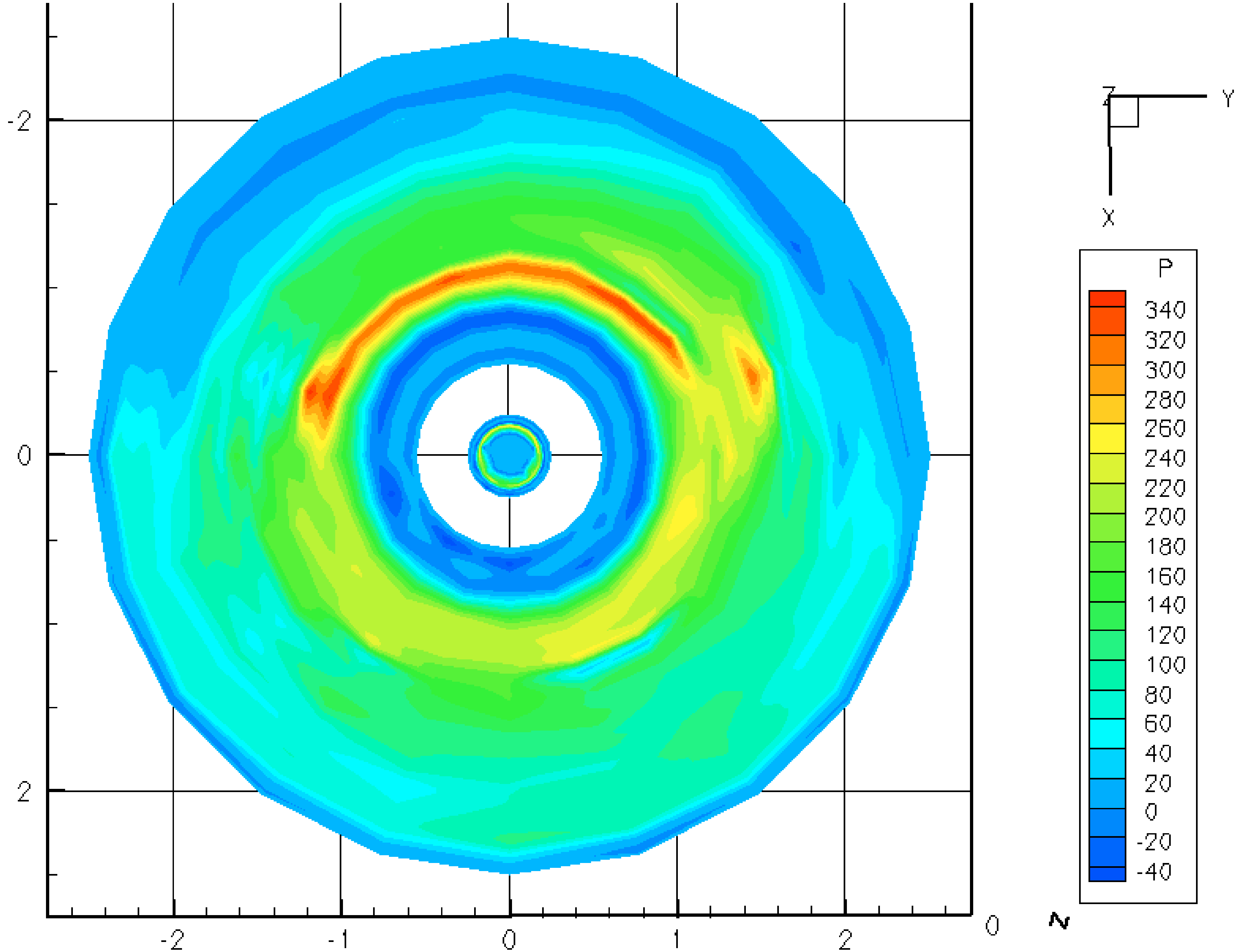
Energy distribution in toroidal modes


2
1
0
-1
-2
-3
E
k̄
0
10
20
30
40
k

Time dependence of parameters at R=1.1 m, Z=0 m


R
e
P
300
250
200
150
100
50
0
0.0
0.5
1.0
1.5
2.0
2.5
t
x10-4
R
e
T
e
120
100
80
60
40
20
0
-20
x1018
R
e
n
1
0
-1
-2
x103
R
e
V
r
1.5
1.0
0.5
0.0
-0.5
-1.0
x10-2
R
e
V
z
x103
R
e
V
p
h
i
-1.5
x10-9
R
e
B
r
R
e
B
z
-0.2
-0.4
-0.6
-0.8
-1.2
R
e
B
p
h
i
8
6
4
2
-2
-4

Pressure contours in midplane in turbulent plasma at t=0.28 ms

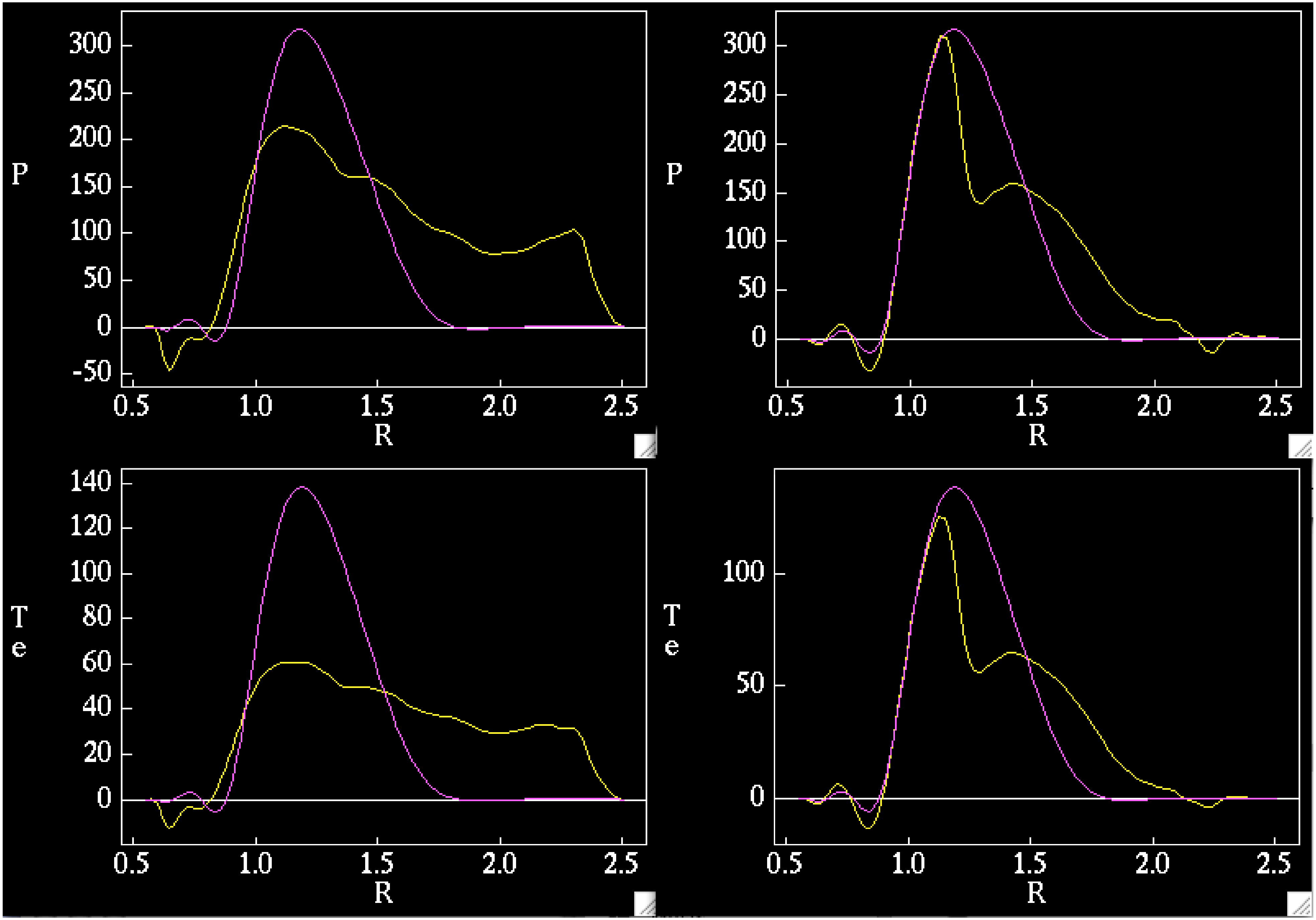


-2
0
2
-2
-1
0
1
2
0
Z
Y
X
Z
P
340
320
300
280
260
240
220
200
180
160
140
120
100
80
60
40
20
0
-20
-40

Midplane pressure profile at two toroidal locations

The n=1 distortion causes the power to spill out at one side ($\phi = 0$).

- Since viscosity eliminates flows near the edge a pedestal forms at $\phi = 0$ with sufficient slope for $\chi_\perp \nabla T$ to transfer the power across the boundary.
- At $\phi = \pi$ the pressure comes up to close to the pre-instability value, (which exceeds the pV^γ limit).


P
300
250
200
150
100
50
0
-50
0.5
1.0
1.5
2.0
2.5
R
P
300
250
200
150
100
50
0
0.5
1.0
1.5
2.0
2.5
R
Te
140
120
100
80
60
40
20
0
0.5
1.0
1.5
2.0
2.5
R
Te
100
50
0
0.5
1.0
1.5
2.0
2.5
R

CONCLUSIONS

- STRONG HEATING CAN DRIVE PEAK PRESSURE A FACTOR 2 ABOVE THE STABILITY LIMIT.

- MHD instability will develop and grow up in $\sim$ 100 μs

  - Higher modes ($n > 20$) will grow first in linear stage

  - In non-linear stage n=1 will dominate leading to large scale particle convection

- MHD will impose a stiff pressure profile close to the marginal profile, $pV^\gamma = constant$.

  - For over-driven case observe a beta collapse leading to close to marginal profile.

  - Results in a turbulent plasma that exhibits rapid particle circulation and enough energy transport to maintain close to ideal pressure profile.

  - Pedestal observed at edge caused by viscous damping of convective flows.

GENERAL COMMENTS

- Non-linear MHD offers a *guess* for the behavior of a strongly driven dipole.
  - Result is dependent on boundary conditions, particularly $\vec{v}$=0.
  - Heating profile together with $\chi_\perp(\vec{r})$ determines pressure profile and therefore stability boundary (NIMROD assumes $\chi_\perp = 1$).

    $\chi_\perp(\vec{r})$ may be determined by micro turbulence.