Session KP1.118

Stability of a Levitated Dipole Confined Plasma in Closed Line magnetic Fields

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- WHY IS DIPOLE INTERESTING?
- RESULTS ON MHD AND DRIFT WAVES
- NEW RESULTS ON CONVECTIVE CELLS
ABSTRACT

The confinement of a plasma in a levitated dipole has been shown to possess uniquely good properties. Ideal MHD indicates (1) equilibria at all beta values, (2) interchange stability when the pressure gradient does not exceed a critical value and (3) ballooning mode stability [1]. The stability of drift frequency modes depends both on the pressure gradient and the plasma profile and these modes are seen to be stable for most interesting profiles. Resistive MHD modes have also been shown to be stable [2]. Theoretical studies indicate that when the critical pressure gradient for interchange stability is exceeded the plasma will develop convective cells that can lead to non-local transport. Pastukhov and Chudin [3] have developed a set of reduced MHD equations and when a critical pressure gradient is exceeded the initial value solutions indicate the formation of convective cells. By assuming a simple flow pattern and then calculating the heating source that balances the convective energy transport we have found steady state solutions to the these equations.

Requirements for “ideal” fusion confinement device.

- MHD instability does not destroy plasma, i.e. no disruptions
- Steady state operation
- High $\beta$ for economic utilization of field
- High $\tau_E$ (before ignition)
  - $\rightarrow$ Ignition in small device
  - $\rightarrow$ Advanced fuel (DD, D-He) possibility
- Low $\tau_p$ for ash removal
- Low divertor heat load:
  - Plasma outside of TF coils $\rightarrow$ large flux expansion.
- Circular, non interlocking coils.

Levitated dipole may fulfill these requirements if physics “works” and technology does not introduce new show-stoppers.
Some Early References


Some Recent Dipole Theory References

(MIT, Columbia, IFS, UCLA, UMd, Kurchatov)

- MHD

- Kinetic theory (Electrostatic)

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• Kinetic theory (Electromagnetic)

• Non-linear
  1d. Tonge, Huang, Leboeuf, Dawson, 2001 APS DPP (LP1059).
• **Between pressure peak and wall**

  MHD stable to interchange when \( \delta(pV^\gamma) \geq 0 \).

  Stable to MHD ballooning when stable to interchange [3a, 4a].

  Can have weak resistive ballooning mode (\( \gamma \propto \gamma_{res} \)) at high beta [4a, Poster KP1.121].

  Stable to electrostatic drift modes when stable to interchange for sufficient \( \eta_j \) [2a, 2b, 2c].

  Electrostatic “entropy” mode essentially unchanged in electromagnetic (high beta) region[2c].

  Unstable interchange modes evolve into convective cells [1d, 2d].

  Convective cells transport particles but not necessarily energy [1d, 2d].

  Convective cells can lead to non-local energy transport with H-mode-like edge [2d].

• **Between Internal Coil and pressure peak (good curvature region)**

  Can have drift modes when \( \nabla n_e \leq 0 \) [2b, 2c].
Stable to all modes when $\nabla n_e > 0$ [2b, 2c].

Can have Drift-cyclotron modes but little energy transport [2d].

Can have convective cells for non-uniform fueling [3d].
Is $B_T$ necessary for toroidal confinement?

- $B_p$ only $\rightarrow$ equilibrium MHD unstable (i.e. FRC)

Two solutions:

(tok, stell, RFP etc) Levitated dipole

Add $B_T \rightarrow$ MHD stable from well and shear.

$\beta << 1$ ($\beta_p \sim 1$) $\beta > 1$ when $p' < p'_{\text{crit}}$

Drifts off flux surfaces $\rightarrow$ MHD stable from compressibility

No drift off flux surfaces

Particles trapped in bad curvature $\rightarrow$ tpm's

No tpm's but drift modes possible near ring.

Important Differences

Magnetic shear $\rightarrow$ Can have convective cells, but without energy transport.

No convective cells

Critical Issues

Divertor Internal ring neutron heat

Steady state Low power density

Disruptions

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Importance of Magnetic Shear

- Closed field line systems lack magnetic shear.

Implications:

MHD

- Shear introduces field-line-bending energy which is stabilizing.

- In equilibrium for an internal ring device force balance established between plasma pressure and field line tension rather than magnetic pressure (field pulls rather than pushes on plasma) → bending energy.

- In sheared fields, near rational surface, field can relax into lower energy state when resistivity is considered.

Electrostatic modes

- Shear is stabilizing for drift waves but:

- In closed field line systems obtain stabilization from plasma compressibility.

- In sheared system no plasma compressibility for passing particles on irrational surfaces.
Interchange Stability: Rosenbluth-Longmire\(\dagger\)

- Closed field line configuration can have “absolute” well when exchange of flux tubes causes internal plasma energy (work+compressibility) to increase.

  Assume adiabatic eq of state: \(p/\rho^\gamma = \text{constant}\).

  \[\Delta E_p = \delta p \delta V + \gamma p \frac{\delta V^2}{V} = \delta S \frac{\delta V}{V}.\]

  \[V = d(Vol)/d\psi = \oint dl/B, S=\text{entropy fct}=pV^\gamma\]

- For \(\delta S > 0\) any exchange of flux tubes will increase plasma energy and damp perturbation.
  - When \(\nabla p/p < \gamma \nabla V/V\), MHD perturbation will damp and vice versa.
  - For \(S = \text{const}\), \(p_{\text{core}}/p_{\text{edge}} = (V_{\text{edge}}/V_{\text{core}})^\gamma\).
  - For Dipole \(p_{\text{crit}} \propto V^{-\gamma} \rightarrow p_{\text{crit}} \propto r^{-20/3}\).
  - Since \(B^2 \propto r^{-6}\), \(\beta = 2\mu_0 p_{\text{crit}}/B^2 \propto r^{-2/3}\) \(\beta\) only decays slowly.

- Microscopically compressibility comes from conservation of adiabatic invariants, \(\mu\) and \(J\).

\(\dagger\) Rosenbluth & Longmire, Ann Phys. \textbf{1} (1957) 120.
MHD: Levitated Dipole

- Consider plasma confined in the field of “floating” ring:
  
  Similar to planetary magnetosphere but field lines close through hole in ring → losses across the field.

- From the point of view of MHD keep in mind:
  
  - No rotational transform, $\vec{B} = \vec{B}_p$ → No shear
  
  - Closed field lines (similar to multipoles)

- Systems with non-rational flux surfaces obtain stability from “average” well and from shear. Dipole stabilized by “compressibility”

**Early Reference:**

MHD Equilibrium

- No rotational transform: $\vec{J} = J_\zeta \hat{e}_\zeta$.

Grad-Shafranov equation becomes:

$$\Delta^* \psi = -\mu_0 R J_\zeta = -\mu_0 R^2 \frac{dp}{d\psi}$$

- Solved by dipole equilibrium code using multi-grid relaxation method for arbitrary beta [1a].

- Analytic solution also found for point dipole and sub critical pressure profile[2a]. (Pressure profile chosen such that $\beta(Z=0)=\text{constant}$.)

Stability of High-n Ballooning Modes

From MHD Energy Principle can show:

* Curvature drive is destabilizing between pressure peak and outer wall.
* Plasma and magnetic field compressibility and bending always stabilizing.

Interchange modes stability requirement:

$$\frac{d\ln p}{d\psi} < 2\gamma \frac{\langle \kappa_\psi \rangle}{1 + \gamma(\beta)}$$
• Minimize $\delta W$ to obtain ODE for ballooning stability.
  (The properties of closed field line balloon eq was discussed by Bernstein et al (1958).)

• **Ballooning stability**
  - For LDX equilibrium at marginal interchange pressure ($p \propto V^{-\gamma}$) and high $\beta$ ($\beta_{\text{max}} \gg 1$) have found that that the lowest order odd mode and all higher modes are stable [Garnier et al].
  - Semi-analytic point-dipole equilibrium with subcritical pressure gradient; stability for $\beta \to \infty$.

• Bernstein (58) showed lowest order even mode stable when the interchange mode is stable. (At marginal stability interchange and ballooning modes coalesce.)

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**Conclusion: Ideal MHD**

• Equilibrium solved analytically and numerically
  
  Dipole exhibits equilibrium all $\beta$.

• Maximum $\beta$ (for a given radial extent of plasma) obtained by choosing equilibria that are marginally interchange stable

• High $\beta$ equilibria found to be stable to high-n ballooning modes.
Convective Cells in large aspect ratio dipole (i.e. hard-core z-pinch)

- **Interchange instability** $\rightarrow$ **Convective cells**
- Pastukhov [24] developed reduced MHD equations that contain long-time-scale, weak, dissipative terms.
- He shows the following:
  - Interchange instability leads to large scale convection with:
    - A broad spectrum azimuthal of modes
    - Time varying solution (i.e. not a steady state)
- Beginning with Pastukhov equations we do the inverse problem.

Assume simple pattern of large scale steady state convective cells. Calculate:
- Implied plasma pressure profile
- Resulting convective energy transport

This yields the heating profile required for the assumed flow pattern.
Pastukhov equations for a hard core z-pinch.

\[ \vec{B} = \vec{e}_\theta B(r) \] included through the Jacobian

\[ J = \pi^{1/2} \oint dl/B = \rightarrow J = B(r)/(2\sqrt{\pi}r) \]

Pressure dependence is simplified through the use of the entropy variable \( \tilde{S} = p/J\gamma \)

Small parameter related transport and the MHD time scales, \( \epsilon^3 \geq \chi/(ac_s) \).

Take \( S(r,z,t) = S(t,r) + \tilde{S}(t,r,z) \) with \( \tilde{S}(t,r,z) \sim \epsilon^2 S(t,r) \).

Assume close to marginality, i.e. \( \rightarrow S' = 0 \).

The adiabatic velocity field to be the \( \mathbf{E} \times \mathbf{B} \) velocity, i.e.

\[ v_a = \frac{1}{J}[\nabla \theta \times \nabla \Phi] \sim (\rho_i/r)c_s \sim \epsilon c_s \quad (1) \]

with \( \Phi \) the electrostatic potential.

Advective terms in the Lagrangian derivatives \( d/dt = (\partial/\partial t + v_a \cdot \nabla) \) give rise to non-linear acceleration terms.

\( d/dt \rightarrow (v_a \cdot \nabla) \) will produce zero-frequency modes that form convective cells.
Energy equation $\rightarrow$ S-equation

- Z-averaged energy equation becomes:

$$\frac{\partial \bar{S}}{\partial t} = 0 \rightarrow$$

$$\frac{c}{r J} \frac{d}{dr} \left( \bar{S} \frac{\partial \Phi}{\partial z} \right) - \frac{\gamma - 1}{2 J^\gamma} \frac{d}{dr} \left( \frac{\gamma - 1}{\rho} \frac{J \bar{S}}{\rho} \right) = \frac{\gamma - 1}{J^\gamma} \bar{Q}_E$$

with $\langle f \rangle \equiv \frac{1}{L} \int_0^L dz \, f(r, z)$

- Marginal profile ($\bar{S} \sim 0$) balances heating ($\bar{Q}_E$) with thermal conduction ($\chi \nabla^2 T$).

- Non-linear term $\langle \bar{S} \Phi \rangle_z$ yields non-local convective-cell transport.
Assuming $\tilde{Q}_E \sim c^3$, the equation for the entropy function fluctuation, becomes:

$$\frac{\partial \tilde{S}}{\partial t} = 0 \rightarrow$$

$$\frac{c}{rJ} \left[ \Phi, \tilde{S} \right] - \frac{c}{rJ} \frac{d}{dr} \left( \tilde{S} \frac{\partial \Phi}{\partial z} \right) + \frac{c}{rJ} \frac{\partial \Phi}{\partial z} \frac{d\tilde{S}}{dr} = \frac{\gamma - 1}{J} \tilde{Q}_E(r,z)$$

(3)

with $[\Phi, f] \equiv \frac{\partial \Phi}{\partial z} \frac{\partial f}{\partial r} - \frac{\partial \Phi}{\partial r} \frac{\partial f}{\partial z}$

Ordinarily when $d\tilde{S}/dr < 0$ we obtain unstable growth described by $\partial \tilde{S}/\partial t \propto -d\tilde{S}/dr$.

Near equilibrium the residual drive can be balanced by the non-linear advection i.e. the instability drive be balanced by a beating of the flow with the pressure perturbation.

• Thus excitation of flows permit an equilibrium with an otherwise unstable pressure profile near to the marginally linearly stable state.

• To solve for $\tilde{S}$ set $\tilde{Q}_E(r, z_0) = 0$. This will determine $\tilde{Q}_E(r, z) = 0$. 

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Following Pastukhov we define a vorticity-like variable:

\[ w = \nabla \theta \cdot (\nabla \times \frac{\rho v_a}{J}) = c \nabla \cdot \left( \frac{\rho \nabla \Phi}{r^2 J^2} \right) = c \nabla \cdot \left( \frac{\nabla \Phi}{c_A} \right) \]  

(4)

with \( c_A \) the Alfvén speed.

Taking \( \nabla \theta \cdot \nabla \times \{ (MHD Momentum Equation)/J \} \) we obtain the resulting steady state vorticity equation as

\[
\frac{\partial w}{\partial t} = 0 \rightarrow c \left[ \Phi, \frac{w}{J} \right] + J^{\gamma - 2} \frac{dJ}{r dr} \frac{\partial \tilde{S}}{\partial z} \approx 0. 
\]  

(5)

The time rate of change of the vorticity is driven by the spatially fluctuating part of the entropy function.
Consider inverse problem - Guess at simple flow pattern:

\[ \Phi(r, z) = \Phi_0 \sin(\pi \frac{r - r_p}{r_w - r_p}) \left( \sin\left(\pi \frac{z}{L}\right) + \sin\left(2\pi \frac{z}{L}\right) \right) \]

Using Eq. (4), \( \Phi(r, z) \rightarrow \) vorticity, \( w(r, z) \).
From Eq. (5) derive the $\tilde{S} = pV^{\frac{1}{3}}$ fluctuation
\[ \langle S \frac{d\phi}{dZ} \rangle \] will determine the excess heating function, \( Q(r) \)
Pressure function, $\bar{S}(r) = \langle pV \rangle$.

Following Eq. (3) we integrate $d\bar{S}/dr$ to obtain $\bar{S}$ (within an integration constant).
Eq (3) also determines the fluctuation in the heating profile, $\tilde{Q}(r,z)$. 
Kinetic Analysis of low-$\beta$ Plasma


- Ideal MHD
  - Assumes adiabatic eq-of-state with $\gamma = 5/3$.
  - Ion FLR and $\eta_i \equiv (n_i \nabla T_i)/(T_i \nabla n_i)$ does not enter single fluid equations.

\[ \text{MHD } d \equiv d \ln p/d \ln (\oint d\ell/B) < \gamma \text{ or } \dot{\omega}_{sp} \leq \gamma \dot{\omega}_{mhd} \]

- There are several interesting orderings:
  - Ideal MHD (short mean free path, collisional)
    \[ \Omega_c > \nu > \omega_b > \omega_s \sim \omega_d \sim \omega \]
  - Long mfp collisional
    \[ \Omega_c > \omega_b > \nu > \omega_s \sim \omega_d \sim \omega \]
  - “Semi-collisional” (Collisionless Ions, Collisional Electrons) expected in LDX
    \[ \Omega_{ce} > \omega_{be} > \nu_e > \omega_{ce} \sim \omega_{de} \sim \omega \]
    \[ \Omega_{ci} > \omega_{bi} > \omega_{si} \sim \omega_{di} \sim \omega > \nu_i \]
  - Collisionless (expected in dipole reactor)
    \[ \Omega_c > \omega_b > \omega_s \sim \omega_d \sim \omega > \nu \]
• From DKE obtain $\tilde{f} = q\phi F_0 e + J_0(k_\perp \rho)h$.

with the non-adiabatic response, $h$, determined from:

$$\left(\omega - \omega_d + iv_\parallel \vec{b} \cdot \nabla'\right) h = - (\omega - \omega_*) q\phi F_0 e J_0(k_\perp \rho) + iC(h).$$

Assuming high bounce frequency the non-adiabatic response $h = h_0$ satisfies

$$(\omega - \omega_d) h_0 = - (\omega - \omega_*) q\phi J_0 F_0 e + iC(h_0) \quad (6)$$

with $\omega_* = \frac{\vec{k} \times \vec{k}_\perp \cdot \vec{b} \times \nabla' F_0}{m_0 F_0}$

$$\omega_d = \vec{k}_\perp \cdot \vec{b} \times \frac{\left(v_\parallel^2 \vec{b} \cdot \nabla' + \nu \vec{B} \cdot \nabla' \right)}{\Omega_c},$$

$$\bar{\phi} = \left(\int \phi \frac{m}{\sqrt{1 - \lambda^2}} \right)/(\int \frac{m}{\sqrt{1 - \lambda^2}}) \quad \text{and} \quad \lambda = \epsilon/\mu.$$

• Dispersion relation: Solve for $h_0$, integrate over velocity space, apply quasi-neutrality.
Long mean-free-path Collisional Regime
(Entropy mode)

- For $\nu_i, \nu_e \gg \omega, \omega_*, \omega_{id}$ obtain $\overline{C}(h_0) \approx 0$. Therefore

$$h_0 = \delta n \left( \frac{m/2n}{T + \delta T} \right)^{3/2} e^{-\epsilon/(T + \delta T)} \approx \left[ \frac{\delta n}{n_0} + \frac{\delta T}{T} \left( \frac{\epsilon}{T} - \frac{3}{2} \right) \right] F_0$$

- Take the flux tube and velocity space average and assume the collision operator conserves particles and energy:

$$\int dl/B \int d^3 v C(h) = \int dl/B \int d^3 v \left( \epsilon - \frac{3}{2} \right) C(h) = 0$$

- For $k_\perp \rho_i \sim 0$ obtain at marginal stability [1b]

$$d = \frac{5}{7} \frac{1 + \eta}{1 - \frac{3}{2} \eta} \quad (7)$$

- $h_1 \sim O(\omega_*/\nu_{ii}) \rightarrow$ “gyro-relaxation” corrections [2b]
Collisionless Ions - Collisional Electrons  
(Semi-Collisional) Regime  

(Likely LDX regime)

Collisionless ion response: From Eq. 1
\[
\frac{\delta n_i}{n_i} = -\frac{q_i}{T_i} \phi + \frac{q_i}{T_i} \int d^3v \frac{\omega - \hat{\omega}_i \epsilon (1 + \eta (c/T_i - 3/2))}{\omega - \hat{\omega}_i (c, \lambda)} \phi F_0 \\
\equiv \frac{q_i}{T_i} (-\phi + \Lambda_i (\omega, \hat{\omega}_i, \hat{\omega}_d))
\]

- Consider particle motion in a point dipole field.
- \( \varpi_{d_i} (\epsilon, \lambda) \) approximation
  \[ \varpi_{d_i} (\epsilon, \lambda) \approx \frac{2}{3} \frac{\epsilon}{T_i} \hat{\omega}_{d_i}. \]
- Include collisional electron response and apply quasi neutrality:
  \[ 2\phi = \Lambda_i (\Omega, d, \eta) + \langle \phi \rangle \Lambda_i^c (\Omega, d, \eta) \]
  with \( \Omega = \omega / \hat{\omega}_{de}, d = \hat{\omega}_{se} (1 + \eta) / \hat{\omega}_{de}. \)
- Obtain solution \( \omega_{crit} = 0.32 \hat{\omega}_{de} \) and
  \[ d = 0.66 \frac{1 + \eta}{1 - 0.51 \eta}. \]  

(8)

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Collisionless Ions and Electrons

- Collisionless dispersion relation

\[
2\phi = \int d^3v \frac{\omega-\omega_{ce}(1+n_i(\frac{e}{m_i}-\frac{1}{2}))}{\omega-\frac{q}{e}\omega_{de}} \phi F_{0e}
\]

\[
+ \int d^3v \frac{\omega+\omega_{ce}(1+n_i(\frac{e}{m_i}-\frac{1}{2}))}{\omega+\frac{q}{e}\omega_{de}} \phi F_{0i}
\]

\[
= \frac{1}{2} (\Lambda_e + \Lambda_{i0}) \int \frac{Bd\lambda}{\sqrt{1-\lambda B}} \phi
\]  

(9)

Taking the flux tube average can obtain \(2 = \Lambda_e + \Lambda_{i0}\)

Substitute into (6), take flux tube avg to obtain:

\[
\oint d\ell B \int \frac{Bd\lambda}{\sqrt{1-\lambda B}} (\phi^2 - \bar{\phi}^2) = \int \tau_0 d\lambda (\phi^2 - \bar{\phi})^2 = 0.
\]

Since \(\phi^2 - \bar{\phi}^2 \geq 0\) obtain flute like, i.e. \(\phi = \phi_0\) to order \(k_\perp^2 \rho_i^2\).

- Following Rosenbluth look for marginality condition

with \(\text{Re}[\omega] = \text{Im}[\omega] = 0\).

\[
d = \frac{1}{3} \left[ \frac{1 + \eta}{1 - \eta} \right].
\]

(10)

- Stability boundary is similar but more restrictive than collisional case.
Conclusions (Drift modes)

- 2 modes are present; MHD-like and drift mode
  - MHD mode stable when \( d < 5/3 \).
  - Drift mode driven by bad curvature \( (d > 0) \) and profile, i.e. \( \eta_j \), effects.
- Collisionality is stabilizing; collisionless modes show larger area of instability.
- Levitated dipole
  - In region between the pressure peak and the wall \( \nabla T < 0 \), \( \nabla n_e < 0 \) and therefore \( \eta_j > 0 \).
  - At the pressure peak \( d = 0 \) and \( \eta_j = -1 \).
  - Between the pressure peak and the internal coil
    LDX: \( \nabla T > 0 \), \( \nabla n_e > 0 \) and \( d < 0 \), \( \eta_j > 0 \).
    Reactor: \( \nabla T > 0 \), \( \nabla n_e < 0 \) and \( d < 0 \), \( \eta_j < -1 \).
Implications for Dipole

- Levitated dipole is uniquely simple and unorthodox approach to plasma confinement.

  Inspired by magnetospheric physics observations. Naturally occurring high-$\beta$ magnetic confinement.

  LDX is first experiment to directly test implications of stabilization by compressibility.

  Test the possibility of near-classical confinement below beta limit and non-local (convective) transport above limit.

  If predictions of high $\beta$ and $\tau_E$ hold up may lead to advanced fuel (D-D or D-He3) fusion.

- Dipole area ripe for innovation:

- Coil set is simple; circular and non-interlocking coils.

- Challenging technology issues:
  - High TC superconducting coil within plasma.
  - Large vacuum chamber $\rightarrow$ low wall loading.

Poster will be available at the LDX web site:

http://www.psfc.mit.edu/ldx/