Helium Catalyzed D-D Fusion in a Levitated Dipole

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Abstract

In plasma containing energetic electrons, two interacting collective modes, an MHD-like mode and a hot electron interchange (HEI) mode [N. A. Krall, Phys. Fluids, 9, 820 (1966)], may be present. The linear stability of interchange modes in a z-pinch at arbitrary beta, including a bulk and hot electron species was recently studied [N. Krasheninnikova, P. J. Catto, Phys. Plasmas, 12, 32101 (2005)]. Using the dispersion relation derived in this reference we show that when necessary conditions are satisfied the two modes may be present or absent in a closed-field line magnetic confinement geometry such as a hard core z-pinch or a dipole. The HEI instability and the MHD-like centrifugally-driven mode have been studied previously [B. Levitt, et al., Phys. Plasmas, 9, 2507 (2002), and 12, 055703 (2005).], including a comparison between the measured mode structure and the predictions of a global low-beta simulation. The radial eigenmode is seen to effect the saturation level of the mode. In the Levitated Dipole Experiment electron cyclotron resonance heating produces high beta plasmas containing hot electrons, and instability observations will be discussed and compared with theoretical predictions.
Outline

- ECRH in LDX
- Instability with hot electrons - linear theory
  - Hot electron interchange (HEI) mode
  - MHD-like background mode
- HEI: nonlinear simulations
  - Chirping of HEI modes
  - LDX Experimental observations
LDX Experiment

- LDX will explore stability & confinement limits in a dipole field.
  - Fusion concept for advanced fuels
- Will operate in either supported or levitated mode.
- Plasma created and maintained by ECH
  - Presently 2.45 and 6.4 GHz. Will add 10.6, 18 & 28 GHz.
  - Creates hot electron population ($T_{eh} > 50$ KeV).
- In addition to MHD modes can be unstable to HEI and loss cone modes.
The Levitated Dipole Experiment (LDX)

MIT/Columbia Research Project
Compressibility can stabilize interchanges

- No compressibility:
  “bad” $\kappa$ & $\nabla B$ drifts causes charge separation -> $V_{ExB}$ increases perturbation

- With compressibility: as plasma moves downwards pressure decreases. For critical gradient there is no charge buildup

In bad curvature pressure gradient is limited to $-\frac{d \ln p}{d \ln V} < \gamma \quad V = \oint dl / B$
Review: MHD Results

- Equilibria exist at high-$\beta$.
- Marginally stable for interchange modes satisfy adiabaticity condition at all $\beta$.
  - \[ \delta(pV^{\gamma}) \equiv \delta(S) = 0, \text{ where } V \equiv \oint \frac{dl}{B}, \quad \gamma = \frac{5}{3} \]
- Stable to ideal MHD ballooning when interchange stable
- No Magnetic Shear -> Convective cells develop when interchange stability violated
  - Keeps $\nabla p$ near critical value.
  - For marginal profiles, convective cells will convect particles but not energy.
    - Leads to have low $\tau_p$ with high $\tau_E$. This property makes LDX particularly interesting for advanced fuels.
LDX has two plasma regions

Bad Curvature region (between pressure peak & vacuum vessel)

- MHD: stable to interchange when $\delta(pV^\gamma)>0$, $V = \oint dl / B$
  - $p_{\text{core}}/p_{\text{edge}} < (V_{\text{edge}}/V_{\text{core}})^\gamma \sim 10^3$ : want large vacuum chamber
    - MHD stability from field bending and not grad-B -> $\beta \sim 1$
    - Unstable interchange modes evolve into convective cells
- Ballooning modes stable when interchange stable
- Weak resistive mode at high $\beta$ ($\gamma \sim \gamma_{\text{res}}$ but no $\gamma \sim \gamma_{\text{res}}^{1/3} \gamma_A^{1/3}$ mode)
- Drift modes: electrostatic “entropy” mode when $\delta(nV)>0$ ($\eta<2/3$), i.e. $n_{\text{core}}/n_{\text{edge}} < (V_{\text{edge}}/V_{\text{core}})$

Good Curvature region (between floating coil and pressure peak)

- “Entropy” (drift) mode can be unstable when $\text{grad}(n_e)<0$
Convective Cells in Dipole

- Convective cells can form in closed-field-line topology
  - Field lines charge up -> $R-\phi$ convective flows
  - 2-D nonlinear cascade leads to large scale vortices
  - Cells circulate particles between core and edge
    - No energy flow when $pV\gamma=\text{constant}$, (i.e. $p'=p'_{\text{crit}}$).
    - When $p'>p'_{\text{crit}}$ cells get non-local energy transport. Only transport sufficient energy transport to maintain $p' \approx p'_{\text{crit}}$.
    - Selective "pumping" at plasma edge can remove tritium.
  - Non-linear calculations use reduced MHD or PIC

Systems with large compressibility

In a closed field line system with a strong field line curvature two basic properties become important in determining the stability of interchange modes. When the plasma pressure varies as

\[ \frac{\partial}{\partial \psi} (pV^\gamma) = 0 \quad \rightarrow \quad d_p \equiv - \frac{d\ln p}{d\ln V} = \gamma \]  

(1)

with \( \psi \) the flux co-ordinate, \( \gamma = 5/3 \) and \( V = \partial (Vol)/\partial \psi = \oint d\ell/B \), the differential flux tube volume, an exchange of flux tubes will not change the pressure profile. Similarly when

\[ \frac{\partial}{\partial \psi} (n_e V) = 0 \quad \rightarrow \quad d_n \equiv - \frac{d\ln n_e}{d\ln V} = 1 \]  

(2)

an exchange of flux tubes does not change the density profile.
ECH in dipole

- ECH heats most strongly where mod-B tangent to field
- Quasi-linear diffusion along ECH characteristics creates radially localized and anisotropic hot electrons.
  - Radially localization yields instability at $f \sim \Omega_{de}$ (conserving $\mu$ and $J$) yielding X-field transport: HEI
  - Anisotropic distribution yields instability at $f \sim \Omega_{ce}$ pitch angle scatter (and loss to supports).
ECRH creates a hot electron species; $T_{eh}>>T_b$

- **Stability similar to EBT (bumpy torus):**
  - MHD-like background mode and kinetic hot electron interchange can both be present.
  - EBT symbiosis -> fragile stability: Background stabilized by diamagnetic well of hot electrons. Hot electron stability requires $n_{eh}/n_b < N_{crit} \sim 0.2$

- **In Dipole, MHD mode stabilized by compressibility**
  - MHD-like mode would lead to convective motion of background which tends to create $n_{core}/n_{edge} \sim V_{edge}/V_{core}$, $p_{core}/p_{edge} \sim (V_{edge}/V_{core})^\gamma$, i.e. to centrally peaked $n_b$ & $p$.
  - In LDX shaping (Helmholtz) coils can vary $V_{edge}/V_{core}$

(Note: ECH is not intrinsic to the dipole concept as it is for EBT)
Hot Electron Interchange Mode (HEI)

- Plasma containing an electron species can be subject to two unstable modes (EBT, ECH mirrors):
  - MHD-like mode feeds on background plasma free energy, $f \sim f_{MHD}$
  - HEI feeds on hot electron density gradient, $f \sim f_{dh}$
- HEI unstable when density gradient exceeds critical value:
  \[ - \frac{d \ln n_{eh}}{d \ln V} < 1 + \frac{m_e^2}{6} \frac{\omega_{dh}}{\omega_{ci}} \frac{n_e}{n_{eh}} \quad with \quad V = \oint \frac{d\ell}{B} \]

Ref: Krall PF 9 (66), Berk PF 13 (70)

Stabilized by background plasma when
\[
\frac{n_{eh}}{n_e} < \alpha, \quad 0.1 < \alpha < 1
\]
- Background mode stability requires
  \[ - \frac{d \ln p_b}{d \ln V} < \gamma \]
- (In presence of loss cone plasma can also generate whistler instability at $f_{ce}$).
LDX exhibits three stability regimes

HEI creates a 3-dimensional space in hot electron fraction, density gradient and mean electron energy.

- **Low density regime**: high $n_{eh}/n_e$ and low $T_{eh}$ limits $d \ln n_{eh}/d \ln V$.
  - ECH maintains peaked & unstable profiles

- **High $\beta$ regime**: Very hot electrons with low $n_{eh}/n_e$.
  - During bursts spreading can stabilize mode

- **Afterglow**: Loss of cold plasma yields high $n_{eh}/n_e$.
  - No ECH so spreading can stabilize mode.
Hot Electron Interchange (HEI) mode

References:

- Krall PF 9 (66), Berk PF 13 (70)
- CTX studies (Low $\beta$ dipole):
  - Warren, Mauel, PRL 74(95), Maslovsky, Leavitt, Mauel PRL90 (2003), …
- Krasheninnikova, Catto, PF 12 (05)32101. Hard-core z-pinch. Hot electrons in finite pressure fluid background at arbitrary $\beta$.

- HEI requires $-\frac{\partial \ln n_{eh}}{\partial \ln V} > 1$ and $\frac{n_{eh}}{n_e} > N_{crit}$
- MHD-like mode requires $-\frac{\partial \ln p_b}{\partial \ln V} > \gamma$
  - In large vacuum chamber of LDX, $n_e$ and $T_{eh}$ gradients criteria can relax.
  - For low $n_b$, the ratio $(n_{eh}/n_e)$ not effected by plasma spreading and HEI is more dangerous

- Several parameters determine stability
- LDX observes HEI and MHD-like mode in high $\beta$ plasma.
High $\beta$, Z-pinch Dispersion Relation

- Arbitrary $\beta$, hard-core z-pinch dispersion relation. Maxwell-hot electrons (Krasheninnikova, Catto, PF 12 (05)32101)

$$
\left(1 + \frac{\beta_b \gamma}{2} - \frac{r \beta_h p'_{0h}}{2p_{0h}} I\right) \left(-\frac{(\frac{dln p_b}{dln V} + \gamma) \omega_{db}^2}{\omega^2} + b + \frac{n_{0h} T_e}{p_{0b}} \left(\frac{T_e G}{T_h} + \frac{(\frac{dln n_{eh}}{dln V} + 1) \omega_{db}}{\omega}\right)\right) \\
+ \frac{\beta_b}{2} \left((1 - H) \frac{n_{0h} T_e}{p_{0b}} + \frac{(\frac{dln p_b}{dln V} + \gamma) \omega_{db}}{\omega}\right)^2 = 0.
$$

Notice appearance of $\frac{dln p_b}{dln V} + \gamma$ and $\frac{dln n_{eh}}{dln V} + 1$

- Limit of $\beta_b$, $T_{eb} \to 0$; $n_{eb}$ finite

$$
\omega_{kh} = \frac{-2 \frac{n_{eh}}{n_b} (1 + \frac{dln n_{eh}}{dln V})(1 - \frac{r \beta_h p'_{0h}}{2p_{0h}} I(\omega))}{2(\frac{n_{eh}}{n_b} G(\omega) + b \frac{T_h}{T_b})(1 - \frac{r \beta_h p'_{0h}}{2p_{0h}} I(\omega)) + \frac{n_{eh} \beta_h}{n_b} (1 - H(\omega))^2}
$$

- $G, H, I$ are complex functions that contain integrals over the hot electron distribution function.
Integrals over hot electron distribution function

The perturbed hot electron densities and currents, \( n_{1h} \) and \( J_{1hr} \) as a function of perturbed fields fields, \( \phi \) and \( B_{1\theta} \) are:

\[
n_{1h}/n_{0h} = \frac{\phi}{T_h} G + \frac{B_{1\theta}}{B_0} H \quad \text{and} \quad (\mu_0 J_{ihr}) = \frac{e\phi}{2T_h} \beta_h H + \frac{B_{1\theta} r\beta_0 p_{0h}'}{2p_{0h}} I
\]

With integrals over velocity \((t^2 = m_e v^2/2T_{eh})\) and pitch angle \((\lambda = v_\parallel/v)\)

\[
G = 1 + \frac{2}{\sqrt{\pi} \omega_{\kappa h}} \int_0^\infty dt \ e^{-t^2} \left( \omega - \omega_{*h} \left[ 1 + \eta_h (t^2 - \frac{3}{2}) \right] \right) \int_{-1}^1 \frac{d\lambda}{\hat{D}(s) - \omega/(\omega_{\kappa h} t^2)}
\]

\[
H = -\frac{2}{\sqrt{\pi} \omega_{\kappa h}} \int_0^\infty dt \ e^{-t^2} \ t^2 \left( \omega - \omega_{*h} \left[ 1 + \eta_h (t^2 - \frac{3}{2}) \right] \right) \int_{-1}^1 \frac{d\lambda(1 - \lambda^2)}{\hat{D}(s) - \omega/(\omega_{\kappa h} t^2)}
\]

\[
-\frac{r\beta_h p_{0h}'}{2p_{0h}} I = -\frac{\beta_h}{\sqrt{\pi} \omega_{\kappa h}} \int_0^\infty dt \ e^{-t^4} \left( \omega - \omega_{*h} \left[ 1 + \eta_h (t^2 - \frac{3}{2}) \right] \right) \int_{-1}^1 \frac{d\lambda(1 - \lambda^2)^2}{\hat{D}(s) - \omega/(\omega_{\kappa h} t^2)}
\]

The t-integrals are evaluated numerically using mathematica.
**Parameter study**

- Can observe both MHD and HEI modes
  - Depends on $\eta = \frac{d \ln T_{eh}}{d \ln n_{eh}}$
- For HEI key parameters are $n_{eh}/n_b$, $T_{he}/T_b$, $\eta$, $\frac{d \ln n_{eh}}{d \ln V}$
- For MHD mode key parameter is $\frac{d \ln p_b}{d \ln V}$

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**Graphs**

- $T_h/T_b = 1e3$, $k_p = 1e-4$, $\eta = 2$, $\beta_h = 0.3$,
  - $\frac{d \ln n_h}{d \ln V} = 0.74$, $\frac{d \ln p_b}{d \ln V} = 3$

- $T_h/T_b = 1e3$, $k_p = 1e-4$, $\eta = 1$, $\beta_h = 0.3$,
  - $\frac{d \ln n_h}{d \ln V} = 1.1$, $\frac{d \ln p_b}{d \ln V} = 3$

- $T_{eh} > 0$ black
Beta affects will modify HEI

- $\beta$ will modify the equilibrium (main effect) and introduce $\tilde{B}$.
- $\beta$-scan: fix background and $n_{eh}/n_e$. Increase $T_{eh}$ and $\beta_h$.
- HEI growth rate asymptotes to higher value.

![Graph showing $\beta_h$ study: $\beta_b=0.015$, $\eta=1$, $n_h/n_0=0.2$, $d\ln p_b / d\ln V=-3$, $k_\rho=1e-4$. HEI (black) and MHD (red).]
HEI stable when \(-\text{dln } n_{eh}/\text{dln } V < 1\)

-\(\text{d ln } p_{eh}/\text{d ln } V\) can exceed \(\gamma\)
- Density (not pressure) gradient determines HEI stability
LDX Operating Regimes

- LDX plasma exhibits 3 operating regimes: high density, high-\(\beta\) and after glow.
Non-Linear HEI simulation

Ref: Warren, Mauel, PRL 74(95),

Flux-Tube Integrated Dynamics
Gyrokinetic Electrons and Cold Ion Fluid Coupled through
2D Electric Fields

Electrons \((F \propto n_e V)\)

\[
\begin{align*}
\dot{\psi} &= \frac{\partial H}{\partial \psi} = \frac{\mu}{e} \frac{c \partial B}{\partial \psi} - \frac{c}{e} \frac{\partial \Phi}{\partial \psi} \\
\psi &= -\frac{\partial H}{\partial \varphi} = \frac{c}{e} \frac{\partial \Phi}{\partial \varphi} \\
\frac{\partial F}{\partial t} + \frac{\partial}{\partial \varphi} (\dot{\Phi} F) + \frac{\partial}{\partial \psi} (\dot{\psi} F) &= 0
\end{align*}
\]

Ions \((N \propto n_i V)\)

\[
\begin{align*}
\frac{\partial N_i}{\partial t} + \frac{\partial}{\partial \varphi} (N_i \|
\nabla \varphi \cdot V\|) + \frac{\partial}{\partial \psi} (N_i \|
\nabla \psi \cdot V\|) &= 0 \\
\frac{\partial N_i}{\partial t} + \frac{\partial}{\partial \varphi} \left[ c N_i \left( \frac{\omega_g(\psi)}{\omega_{ci} B} - \frac{\| \nabla \psi \|^2}{\omega_{ci} B} \frac{\partial^2 \Phi}{\partial \varphi^2} \right) \right] + \frac{\partial}{\partial \psi} \left[ c N_i \left( \frac{\partial \Phi}{\partial \varphi} - \frac{\| \nabla \psi \|^2}{\omega_{ci} B} \frac{\partial^2 \Phi}{\partial \psi \partial t} \right) \right] &= 0
\end{align*}
\]

\[
\frac{\partial}{\partial \varphi} \left( h_{\varphi} \frac{\partial \Phi}{\partial \varphi} \right) + \frac{\partial}{\partial \psi} \left( h_{\psi} \frac{\partial \Phi}{\partial \psi} \right) = -4\pi e (N_i - N_e)
\]

Electric Potential
Summary

- Hot electron pressure gradient can exceed the MHD limit. **Consistent with experiments.**
- Background plasma limited by MHD. Expected to exhibit convective flows when critical pressure gradient exceeded in background plasma.
- Non-linear simulations reproduce observed mode chirping.

- **Exploration of ECH plasmas progressing in LDX**

Website: www.psfc.mit.edu/ldx/
**Flux-Tube Integrated Dynamics**

**Gyrokinetic Electrons and Cold Ion Fluid Coupled through 2D Electric Fields**

**Electrons (F ∝ n_eV)**

\[
\begin{align*}
\dot{\phi} &= \frac{\partial H}{\partial \psi} = \mu \frac{c \partial B}{e \partial \psi} - c \frac{\partial \Phi}{\partial \psi} \\
\dot{\psi} &= -\frac{\partial H}{\partial \varphi} = c \frac{\partial \Phi}{\partial \varphi} \\
\frac{\partial F}{\partial t} + \frac{\partial}{\partial \varphi}(\dot{\phi} F) + \frac{\partial}{\partial \psi}(\dot{\psi} F) &= 0
\end{align*}
\]

**Ions (N ∝ n_iV)**

\[
\begin{align*}
\frac{\partial N_i}{\partial t} + \frac{\partial}{\partial \varphi} \left( N_i \|\nabla \varphi \cdot \mathbf{V} \| \right) + \frac{\partial}{\partial \psi} \left( N_i \|\nabla \psi \cdot \mathbf{V} \| \right) &= 0
\end{align*}
\]

**Gravity**

\[
\begin{align*}
\frac{\partial N_i}{\partial t} + \frac{\partial}{\partial \varphi} \left[ c N_i \left( \omega_g(\psi) - \frac{\partial \Phi}{\partial \psi} - \frac{\|\nabla \varphi\|}{\omega_{ci} B} \frac{\partial^2 \Phi}{\partial \varphi \partial t} \right) \right] + \frac{\partial}{\partial \psi} \left[ c N_i \left( \frac{\partial \Phi}{\partial \varphi} - \frac{\|\nabla \psi\|^2}{\omega_{ci} B} \frac{\partial^2 \Phi}{\partial \psi \partial t} \right) \right] &= 0
\end{align*}
\]

**Electric Potential**

(Constant along B-line & small dissipation)

\[
\frac{\partial}{\partial \varphi} \left( h_{\varphi} \frac{\partial \Phi}{\partial \varphi} \right) + \frac{\partial}{\partial \psi} \left( h_{\psi} \frac{\partial \Phi}{\partial \psi} \right) = -4\pi e (N_i - N_e)
\]
Self-Consistent, Nonlinear, **Multi-Fluid, Field-Line Integrated**, Simulation Reproduces Dipole Interchange Dynamics and Mode Structure

**Dynamics: Frequency Rise**

**Structure: Broad, Multi-Mode**
Dipole interchange modes have **Broad** radial structures.

**Centrifugal Interchange**

- **Subplot a)**
  - m=1
  - Graph showing correlation amplitude vs. radius (cm).

- **Subplot b)**
  - m=2
  - Graph showing correlation amplitude vs. radius (cm).

- **Subplot c)**
  - m=3
  - Graph showing correlation amplitude vs. radius (cm).

**Hot Electron Interchange**

- **Subplot a)**
  - m=1
  - Graph showing normalized correlation amplitude vs. radius (cm).

- **Subplot b)**
  - m=2
  - Graph showing normalized correlation amplitude vs. radius (cm).

- **Subplot c)**
  - m=3
  - Graph showing normalized correlation amplitude vs. radius (cm).

(2D Poisson’s Equation: Computed mode structure shown with solid lines.)
Dipole Plasma Confinement

- Toroidal confinement without toroidal field
  - Stabilized by plasma compressibility
  - No neoclassical effects
  - No TF or interlocking coils
  - $p'$ constraint -> small plasma in large vac vessel

- Poloidal field provided by internal coil
  - Steady-state w/o current drive
  - $J_\parallel = 0$ -> no kink instability drive

If $p_1 V_1^\gamma = p_2 V_2^\gamma$, then interchange does not change pressure profile.

For $\eta = \frac{d \ln T}{d \ln n} = \frac{2}{3}$, density and temperature profiles are also stationary.
Floating Coil Produces Strong Dipole Field