BP1.93

Basic Physics in the Levitated Dipole Experiment

J. Kesner, L. Bromberg
Plasma Science and Fusion Center, MIT
Cambridge, Ma. USA

D.T. Garnier and M.E. Mauel
Department of Applied Physics, Columbia University
New York, N.Y. USA

Abstract

The LDX experiment will test MHD equilibrium and stability conditions as well as transport issues in a levitated dipole configuration. In the levitated dipole concept the toroidal field is zero and the curvature is in the poloidal plane. The drifts and diamagnetic currents are expected to be toroidal.

The equilibrium is determined from the solution of the Grad-Shafranov equation. MHD stability dictates that the interchange stability is marginal when $pV^\gamma = constant$ with $V = \oint dl/B$. By requiring the pressure profile be marginal (up to the imposed position of the pressure peak) we can obtain equilibria with the maximum allowed pressure gradient and the highest stable peak pressure. We have examined the ballooning stability of these profiles and found them to be stable, even for equilibria with a peak local $\beta >> 1$. The ballooning stability comes about because as the pressure increases the plasma spreads out in major radius and the pressure gradient decreases in the region of enhanced curvature near the outer midplane.

Also, closed field line systems like a levitated dipole can exhibit convective cells when heating is not axisymmetric. We have examined the consequences of convective plasma flows.

Requirements for "ideal" fusion confinement device.

- MHD instability does not destroy plasma, i.e. no disruptions
- Steady state operation
- High β for economic utilization of field
- High τ_E

  Ignition in small device

  Advanced fuel (DD, D-He) possibility
- Low τ_p for ash removal
- Low divertor heat load:

  Plasma outside of TF coils $\rightarrow$ large flux expansion.
- Circular, non interlocking coils.

Levitated dipole may fulfill these requirements if physics "works" and technology does not introduce new show-stoppers.

Levitated Dipole idea was inspired by magnetospheric research

Jovian magnetosphere discovered by Voyager I, II and Galileo spacecraft. $\beta > 1$ fueled by solar wind and by Io volcanism.

Io plasma torus is example of "pellet" fueling.

Application for fusion confinement first suggested by Hasegawa (87).

Some Early References

A. Hasegawa, Comments on Plasma Physics and Controlled Fusion, **1**, (1987) 147.

A. Hasegawa, L. Chen and M. Mauel, Nuclear Fus. **30**, (1990) 2405.

E. Teller, A. Glass, T.K. Fowler et al., Fusion Technology **22**, (1992) 82.

Levitated dipole; a new approach for fusion research.

- Physics different and complimentary to tokamak.
- Inspired by understanding of magnetospheric plasma physics.
- Physics basis in mirror, multipole and levitron fusion research.
- May satisfy criteria for “ideal” reactor if technology does not create show-stoppers.

  Internal ring in (advanced fuel) fusing plasma is a challenge.

  Tends to have low power density at first wall.

Is B_T necessary for toroidal confinement?

- B_p only: equilibrium but MHD unstable (i.e.FRC)

Two solutions:

(tok, stell, RFP etc)	**Levitated dipole**
Add $B_T \rightarrow$ MHD stable from well and shear.	Levitated ring $\rightarrow$ MHD stable from compressibility
$\beta << 1 \quad (\beta_p \sim 1)$	$\beta \sim 1 \;\; p' < p'_{crit}$
Drifts off flux surfaces $\rightarrow$ NC effects	No drift off flux surfaces
particles trapped in bad curvature $\rightarrow$tpm's	No tpm's

Important Difference

Magnetic shear $\rightarrow$ No convective cells	Can have convective cells.

Some Difficulties

Small flux expansion, steady state, disruptions	Internal Ring, Low power density

Interchange Stability; Rosenbluth-Longmire†

- Closed field line configuration can have "absolute" well when exchange of flux tubes causes internal plasma energy (work+compressibility) to increase.

  Assume equation of state: $p/\rho^\gamma = f(\psi)$.

  $\Delta E_p = \delta p \delta_V + \gamma p \frac{\delta V^2}{V} = \delta V \delta(pV^\gamma)/V^\gamma$.

  with $V = d(Vol)/d\psi = \oint dl/B$

- For $\delta(pV^\gamma) > 0$ any exchange of flux tubes will increase plasma energy and damp perturbation.

  When $\nabla p/p < \gamma \nabla V/V$, MHD perturbation will damp and vica versa.

  $p_{core}/p_{edge} = (V_{edge}/V_{core})^\gamma$.

  For Dipole $p_{crit} \propto V^{-\gamma} \rightarrow p_{crit} \propto r^{-20/3}$.

  Since $B^2 \propto r^{-6}$, $\beta = 2\mu_0 p_{crit}/B^2 \propto r^{-2/3}$ only decays slowly.

(Microscopically compressibility comes from conservation of adiabatic invariants, μ and J.)

† Rosenbluth and Longmire, Ann Phys. **1** (1957) 120.

Levitated Dipole Experiment, LDX

$$\frac{p_{core}}{p_{edge}} = \left[\frac{\oint_{edge} dl/B}{\oint_{core} dl/B}\right]^{\gamma}$$

- Design rule: need large flux expansion.

  $\rightarrow$ small ring in large vacuum chamber.

  For LDX $r_{ring} = 40\ cm,\ R_{vac} = 2.5\ m$
  $10^3 < p_{core}/p_{edge} < 10^4$.

  (Compare with a levitron).

- Physics goal of LDX

  * Study high-β plasma stabilized by compressibility.

  * When do convective cells form in shear-free configuration?

  - Relationship between MHD stable profiles and the elimination of drift waves.

  - Coupling between SOL and core plasmas.

  - Long time evolution of high beta dipole confined plasma.

  - Stability of high-β energetic particles in dipole magnetic fields.

LDX Experiment

- Joint MIT/CU project
- Located in Tara cell at MIT PSFC.
- 5 year program. First plasma: Fall 2000.

MHD: Levitated Dipole

- Consider plasma confined in the field of “floating” ring:

  Similar to planetary magnetosphere but field lines close through hole in ring so losses have to be across the field.

- From the point of view of MHD keep in mind:

  No rotational transform, $\mathbf{B} = \mathbf{B}_p$.

  No shear

  Closed field lines (like multipoles)

- Systems with ergodic (non-rational) flux surfaces obtain stability from “average” well and from shear.

  Early MHD References:

  Rosenbluth and Longmire, Ann Phys. **1** (1957) 120.

  Bernstein, Frieman, Kruskal, Kulsrud, Proc. R. Soc. London, Ser. A, **244** (1958) 17.

MHD Formulation - Equilibrium

Ref: D. Garnier, J. Kesner, M. Mauel, Phys Pl **6**, 3431 (1999)

- No rotational transform: $\vec{J} = J_\zeta \vec{e}_\zeta$.

  Use notation from J.P. Freidberg, "Ideal Magneto-Hydrodynamics", Plenum Press (1987).

  Grad-Shafranov equation becomes:

$$\triangle^* \psi = -\mu_0 R J_\zeta = -\mu_0 R^2 \frac{dp}{d\psi}$$

- Solved by dipole equilibrium code using multi-grid relaxation method for arbitrary beta [Garnier et al].

  Typical solutions found for $\beta = 0$ (vacuum field) and $\beta_{max} = 10$. ($\beta \equiv 2\mu_0 p / B^2$ is local β).

- Analytic solution for point dipole and special pressure profile, in Krasheninnikov et al, PRL **82** (1999) 2689.

Convective Cells in a Closed Field line Configuration

- With closed field lines pressure and electric potential tend to be constant on field lines. This leads to equilibrium variation in flux and toroidal angle.

  With ergotic flux surfaces only variation is in flux.

- Heating asymmetry→pressure asymmetry → convective flows.

- $E \times B$ drift expected to be comparable to diamagnetic drift.

  Cannot use MHD flow: $v_\perp = \vec{E} \times \vec{B}/B^2$.

  Use drift model which includes both diamagnetic and electric field ($E \times B$) flows.

- Some references from previous theoretical and experimental work.

  - Dawson and Okuda [Dawson et al., PRL **27** (1971) 491] pointed out that convective cells can form from excitation of stable modes.

  - Convective cells have been observed to form in closed field line Octupoles [Navratil et al, PF **20** (1977) 157].

Drift Model Equations†

In the drift model we approximate the flow by:

$$\vec{V} = \frac{\vec{B} \times \nabla\phi}{B^2} + \frac{\vec{B} \times \nabla p_i}{enB^2} + V_{\|}\vec{b} \equiv \vec{V}_{E\times B} + \vec{V}_{\nabla P} + V_{\|}\vec{b}$$

and $V_{E\times B} \sim V_{\nabla p}$. Fluid equations take form:

$$\nabla(\rho\vec{V}) = S$$

$$\rho\vec{V} \cdot \nabla\vec{V}_{E\times B} + \rho(\vec{V}_{E\times B} + V_{\|}\vec{B}) \cdot \nabla\vec{V}_{\|} = -\nabla p + \vec{J} \times \vec{B}$$

* In advective derivative $\vec{V} \rightarrow \vec{V}_{E\times B}$ due to gyro-viscous cancellation.

Use orthogonal flux co-ordinates $(\psi,\ \zeta,\ \chi)$ such that $\vec{B} = \nabla\psi \times \nabla\zeta = \alpha\nabla\chi$.

† Hazeltine & Meiss, Addison-Wesley (1992).

Make Simplifying assumptions:

- Use B-field of wire loop →elliptic functions. Assume low β.
- Equilibrium axisymmetric to lowest order.
- Linearize, $p/p_0 \ll 1$, etc. and Fourier analyze.

$$p(\psi,\zeta) = p_0 + p(\psi)cos(n\zeta)$$

$$\phi(\psi,\zeta) = \phi_0 + \phi(\psi)cos(n\zeta)$$

- Assume up-down symmetry. At midplane:

  $V_{\|}\partial/\partial\chi = 0$,

  $d\psi = -BRdR$, can use (R,ζ) coordinates.

Obtain coupled ODEs for ϕ and B_{1Z}.

- The R and the ζ components of the momentum eq. become:

$$R\phi'\left(2\phi_0' + \frac{p_0'}{en}\right) - n^2\phi\left(\phi_0' + \frac{p_0'}{en}\right) + R\phi_0'\frac{p'}{en} =$$

$$\frac{(RB_0)^2}{\rho}\left((p' + \frac{B_0B_{1Z}' + B_0'B_{1Z}}{2\mu_0}) - 2\vec{\kappa}_R\frac{B_0B_{1Z}}{\mu_0}\right)$$

and

$$\phi'\left(\phi_0' + \frac{p_0'}{en}\right) - \phi\left(\frac{\phi_0'}{R} + \frac{p_0'}{enR} + \phi_0'' - \phi_0'\frac{dlnB_0}{dR}\right) -$$

$$\phi_0''\frac{p}{en} + \phi_0'\frac{p}{en}\frac{dlnB_0}{dR} = -\frac{B_0^2}{\rho}\left(p + 2\frac{B_0B_{1Z}}{2mu_0}\right)$$

with $dp/dR \equiv p'$, etc.

Consider these as equations for ϕ and B_{1Z}.

- To solve choose pressure and density profiles:

$$p_0(R) = 0.5 p_{00} \left(1 - cos(2\pi \frac{R - R_0}{R_w - R_0})\right)^{10}$$

$$\rho(R) = m_i n(R) = 0.5 \rho_0 \left(1 - cos(2\pi \frac{R - R_0}{R_w - R_0})\right)^{8}$$

- Choose a rigid rotation for ϕ_0 profile.
- Pressure asymmetry.

  For heating asymmetry $H(\psi, \zeta) = H_0(\psi) + \delta H(\psi, \zeta)$ pressure asymmetry becomes $\delta p/p \sim (1/\tau_E \omega_\zeta) \delta H/H$.

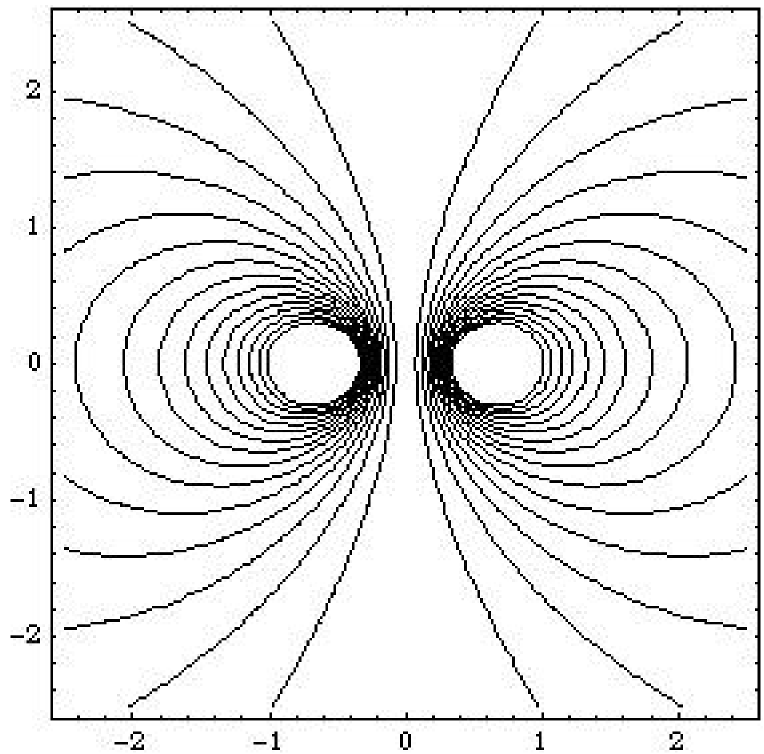
  - For $\delta H/H \sim 5\%$ get $p/p_0 \sim 0.005$

- Large change in ϕ occurs in region where fluid flow stagnates, i.e.

$$\phi_0' + \frac{p_0'}{en} \sim 0$$

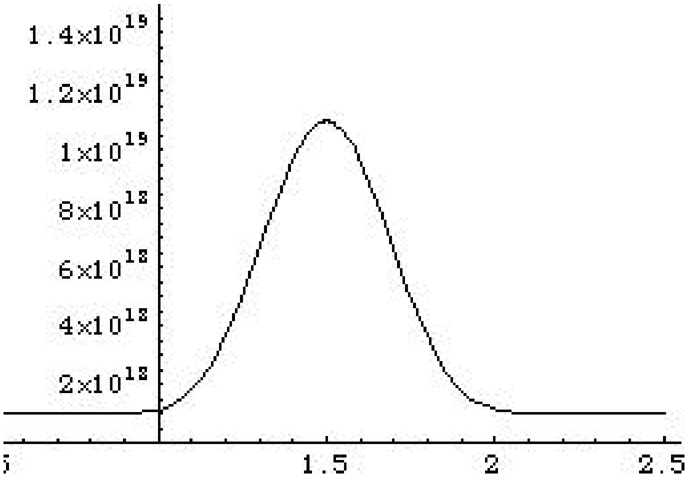
  - This effect only observed in drift model.
  - Convective cells tend to form in outer (beyond pressure peak) region

Field of a Current Loop (R_{loop}=0.5 m)

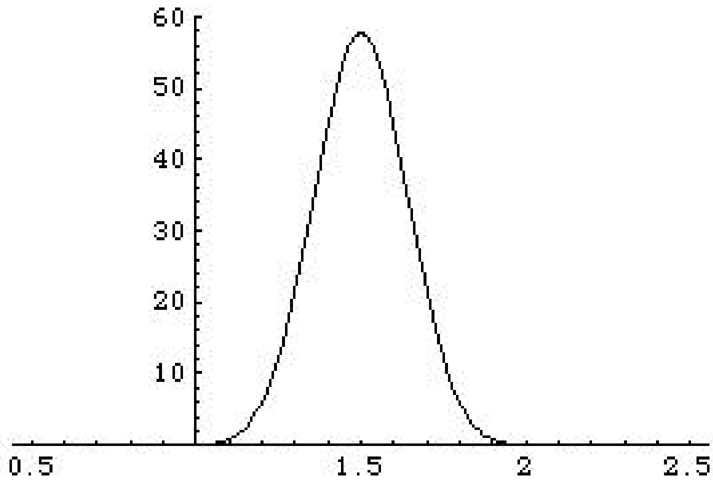


Model Density and Pressure profiles

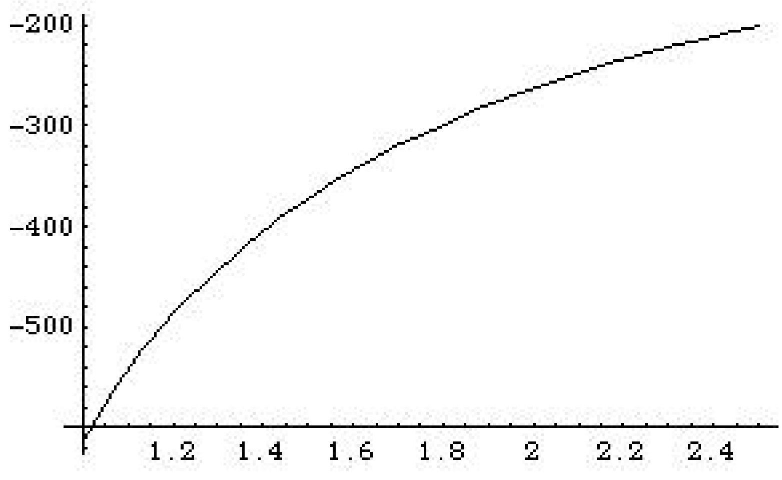
$n_0(R)$ (m^{-3})



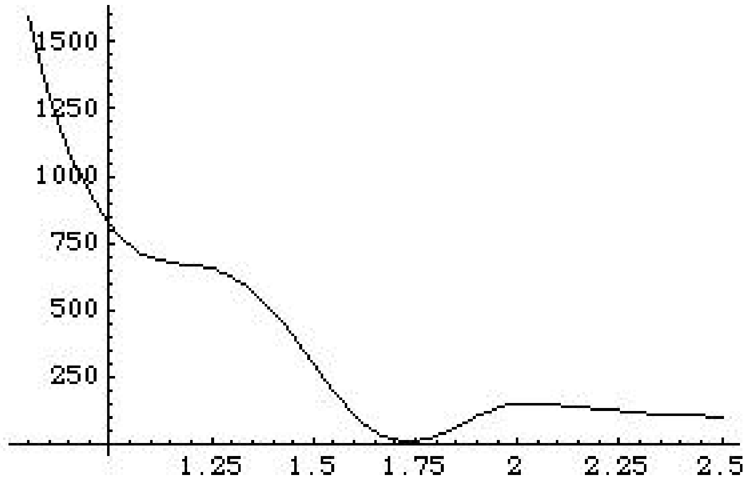
$P_0(R)$ (Pa)



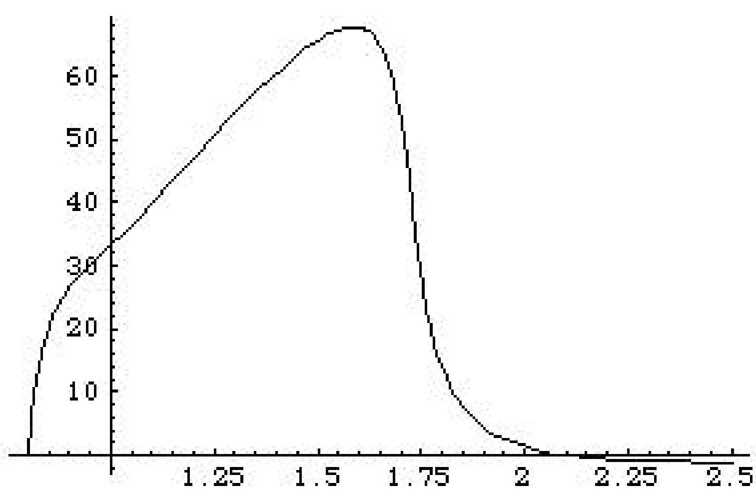
Assume Rigid Rotation Electrostatic potential $\varphi_0(R)$



Sum of ExB and grad(p) drifts

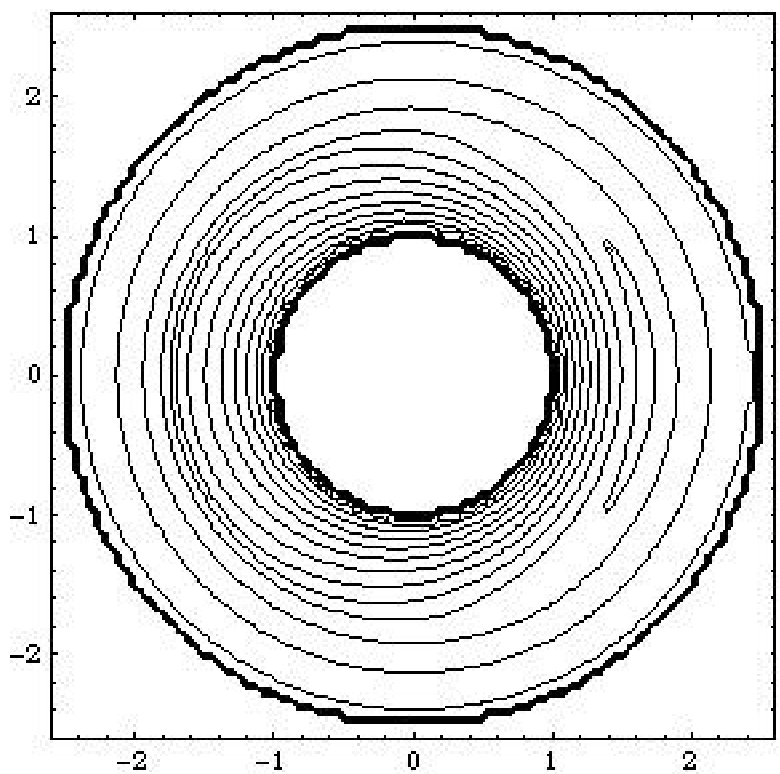


Midplane Potential, $\varphi(R)$, vs Radius



Midplane potential for $p(R,\zeta)=\delta p(R)\cos(2\pi\zeta)$

$\delta p/P_0=0.005$



Conclusions on Flow

- Convective cells will form when there is asymmetric pressure.
  - Heating asymmetry → pressure assymetry.
  - Convective cells are an example of equilibrium with flows.
  - Cells tend to form in region where fluid flow is small.
- For marginal interchange stability convective flows transport particles, but not energy.

  May provide fueling mechanism in advanced fuel reactor.

Stability of High-n Ballooning Modes

From JPF we can write:

$$\delta W_F = \frac{1}{2\mu_0}\int_p d^3r\Big[|\vec{Q}_\perp|^2 + B^2|\nabla\cdot\vec{\xi}_\perp + 2\vec{\xi}_\perp\cdot\vec{\kappa}|^2$$

$$+\gamma\mu_0 p\langle\nabla\cdot\vec{\xi}_\perp\rangle^2 - (2\mu_0\vec{\xi}_\perp\cdot\nabla p)(\kappa\cdot\vec{\xi}^*_\perp)\Big]$$

$\vec{Q} = \nabla\times(\vec{\xi}\times\vec{B})$ and $\vec{\xi}_\perp$ is the amplitude of the perpendicular displacement.

The flux average defined as: $\langle c\rangle = \oint c\ (d\ell/B)/\oint d\ell/B$

- The plasma compression term derives from the closed field line periodicity constraint which yields $\nabla\cdot\vec{\xi}\rightarrow\langle\nabla\cdot\vec{\xi}_\perp\rangle$.
- We can minimize the sum of the stabilizing magnetic+plasma compression terms to obtain

$$B^2|\nabla\cdot\vec{\xi}_\perp + 2\vec{\xi}_\perp\cdot\vec{\kappa}|^2 + \gamma\mu_0 p\langle\nabla\cdot\vec{\xi}_\perp\rangle^2 \rightarrow \frac{4\langle\vec{\xi}_\perp\cdot\vec{\kappa}\rangle^2}{1+\gamma\langle\beta\rangle/2}.$$

High-n Modes

Approximate $\vec{\xi}_\perp = \vec{\eta}_\perp e^{iS}$, where $\nabla S \equiv \vec{k}_\perp$ and $\vec{B} \cdot \nabla S = 0$.

Following JPF we can obtain to lowest order (in $1/k_\perp a$), $\vec{\eta}_{\perp 0} = (X/B)\vec{b} \times \vec{k}_\perp$.

Flux coordinates: $\vec{B} = \nabla\psi \times \nabla\zeta = \alpha\nabla\chi$

The second order contribution to δW then becomes

$$\delta W_2 = \frac{1}{2\mu_0} \int d\psi d\zeta \int J d\chi \Big[\frac{k_\perp^2}{J^2 B^2} \left(\frac{\partial X}{\partial \chi} \right)^2$$

$$-2\mu_0 \left(\frac{\partial S}{\partial \zeta} \right)^2 \frac{dp}{d\psi} \kappa_\psi X^2 + 4\gamma\mu_0 p \left(\frac{\partial S}{\partial \zeta} \right)^2 \frac{\langle \kappa_\psi X \rangle^2}{1 + \gamma \langle \beta \rangle / 2} \Big].$$

* Curvature drive destabilizing when $\kappa_\psi < 0$.
* Compressibility, bending always stabilizing.

- Minimize δW to obtain marginal stability ODE:

$$B\frac{d}{d\ell}BR^2\frac{dX}{d\ell}+2\mu_0\kappa_\psi p_\psi X-4\gamma\mu_0 p\kappa_\psi\frac{\langle X\kappa_\psi\rangle}{1+\gamma\langle\beta\rangle/2}=0$$

with $p_\psi = dp/d\psi$ & $\kappa_\psi = \kappa\nabla\psi$.

Ballooning Equation

$$B\frac{d}{d\ell}BR^2\frac{dX}{d\ell}+2\mu_0\kappa_\psi p_\psi X-\frac{4\gamma\mu_0 p\kappa_\psi\left\langle X\kappa_\psi\right\rangle}{1+\gamma\left\langle\beta\right\rangle/2}=0$$

- Consider interchange modes: X=constant

$$\frac{2\gamma p\langle\kappa_\psi\rangle}{1+\gamma\langle\beta\rangle/2}\geq p_\psi$$

- Consider flux tube average to obtain a constraint.

$$\langle X\kappa_\psi\rangle\left[\frac{2\gamma p\langle\kappa_\psi\rangle}{1+\gamma\langle\beta\rangle/2}-p_\psi\right]=0$$

Thus when interchange modes are stable the solution requires $\langle X\kappa_\psi\rangle=0$. Balloon Eq. becomes:

$$B\frac{d}{d\ell}\frac{1}{BR^2}\frac{dX}{d\ell}+2\alpha\mu_0\kappa_\psi p_\psi X=0$$

$\langle X\kappa_\psi\rangle=0$ can lead to both odd and even solutions but odd solution tends to be more unstable.

Radial profiles of B_{min} and p(R) ($\beta_{max} = 10$)

- Maximum pressure gradient chosen so $p(\psi)$ satisfies marginal interchange stability.
- Equilibrium found with maximum local β, $\beta_{max} = 10$, flux average β, $\langle\beta\rangle_{max} = 3.9$ and volume average β, $\overline{\beta} \sim 0.47$.

High-β Equilibrium.

Quasi-Eigenmodes ($\beta_{max} = 10$, line 5)

- All field lines are stable to ballooning.

  Origin of stability: Odd mode has zero amplitude in region of large curvature and high β.

Conclusions: MHD

- Dipole exhibits very high β equilibria.
- Maximize β (for a given radial extent of plasma) obtained by choosing equilibria that are marginally interchange stable

  What happens when ∇p exceeds critical value?

- High β equilibria found to be stable to high-n ballooning modes.
- Large vacuum chamber required to permit large flux expansion but: size can be reduced with edge pressure pedestal. Consider:
  - Stabilizing effect of flow shear.
  - Stabilizing effect of magnetic limiter.

General Conclusions

- Levitated dipole is uniquely *simple* and *unorthodox* approach to plasma confinement.

  Inspired by magnetospheric physics observations. Naturally occurring high-β magnetic confinement.

  LDX is first experiment to test stabilization by compressibility.

  If predictions of high β and τ_E hold up may lead to a new fusion scheme for advanced fuel plasmas.

- Area ripe for innovation:

  - Convective cells could fuel plasma,

  - Edge transport barrier would be big win.

- Challenging technology issues - but coil set is simple; circular and non-interlocking coils.

Poster will be available at the LDX web site:

http://www.psfc.mit.edu/ldx/