Reconstruction of Pressure Profile Evolution during Levitated Dipole Experiments

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Abstract

Magnetic levitation of the LDX superconducting dipole causes significant changes in the measured diamagnetic flux and what appears to be fascinating temporal evolution of plasma diamagnetic current. This poster describes the reconstruction of plasma current and plasma pressure profiles from external measurements of the equilibrium magnetic field, which vary substantially as a function of time. Previous free-boundary reconstructions of plasma equilibrium [1] showed the plasma to be anisotropic and highly peaked at the location of the cyclotron resonance of the microwave heating sources. Reconstructions of the peaked plasma pressures confined by a levitated dipole incorporate the small axial motion of the dipole (+/− 5 mm), time varying levitation coil currents, eddy currents flowing in the vacuum vessel, constant magnetic flux linking the superconductor, and new flux loops located near the hot plasma in order to closely couple to plasma current and dipole current variations.

Key Points

- During magnetic levitation, vertical motion of the superconducting dipole, changes in the levitation control coil current, and induced eddy currents couple to magnetic diagnostics.

- We “self-calibrate” the magnetics using levitation current ramps and pre-programmed “jogs” of dipole’s vertical position.

- Using data from the calibration shots, the induced eddy currents are calculated (and digitally “removed”) by inverting coupled linear ODEs. This allows...

- Use of previous magnetic reconstruction methods.
Magnetic Detectors

The magnetic field is axisymmetric. The dipole motion is axial. The \( i \)th magnetic detector, whether a flux coil or a magnetic field vector component, can be designated by a location \((R_i, Z_i)\). The magnetic signal detected by the \( i \)th detector is given by the equation

\[
S_i(t) - S_i(0) = G_{i,L}[I_L(t) - I_L(0)] + \sum_w G_{i,w}I_w(t) \\
+ [G_{i,D}(t)I_D(t) - G_{i,D}(0)I_D(0)] + \sum_p G_{i,p}(t)I_p(t)
\]  \( (1) \)

In Eq. 1, the signal equals the sum of the response from all coupled equilibrium, control, and vessel eddy currents in proportion to a Green’s function, or mutual inductance. \( I_L \) is the levitation control current, \( I_w \) are the eddy currents flowing in the vessel, \( I_D \) is the dipole current, and \( I_p \) are the plasma ring currents.
Reconstruction

The wall eddy currents can be expressed as an expansion of orthogonal, and axisymmetric, “modes” with decreasing current decay times.

Since the flux linked by the superconducting dipole is constant, \( I_D(t) \) can be determined simultaneously with the solution to Eq. 1 with knowledge of the dipole’s axial position. The constant flux constraint is

\[
0 = L_D I_D(t) + \sum_p M_p I_p(t) + M_L(t) I_L(t) + \sum_w M_w(t) I_w(t).
\]

Eqs. 1 and 2 represent a set of simultaneous linear equations for the unknown currents, \((I_p, I_D, I_w)\). Using only 15 working flux loops, the number of unknown currents could reach 15. In practice, some of the flux loop measurements are not independent, and the number of unknown currents must be much smaller and determined by practice. A good choice should be two or three “plasma” current rings and one or two “wall eddy current modes.” With \( p \in \{1, 2\} \) and \( w \in \{1\} \), there are four current unknowns including \( I_D \). The least squares most likely values of these four currents can be determined from the 15 flux loops using singular value decomposition (SVD).
Outline

(1) Previous magnetic reconstruction results from LDX and space

(2) “Self calibration” using levitation control and dipole position ramps

(3) How much current in the superconducting dipole? Measurement using weight

(4) Estimating the diamagnetic plasma current

(5) Initial plasma pressure reconstruction. Best fit pressure is isotropic during levitation!
(1) Previous Results

- High beta plasmas created like those found in magnetosphere
- Anisotropic
- Required x-ray imaging to determine peak pressure
- Ring current \( \sim \) Plasma Stored Energy 
  \( W_p \approx 170 \text{ (J/kA)} \ I_p \)
Ring Current:  
Trapped, High-$\beta$ Protons (15-250 keV)

- Greatly intensified during geomagnetic storms
- $T_i \sim 7T_e$ and $P_\perp \sim 1.5 P_{||}$
- Monthly storms: $\sim 5$ MA. (LDX: 3-4 kA) 10 MA storms few times a year.
- Current centered near $L \sim 4-5R_e$; $\Delta L \sim 2.6R_e$ wide and $\Delta z \sim 1.6R_e$; Not axisymmetric.
- Curlometer during storms: $J_{RC} \sim 25$ nA/m$^2$ (Cluster II, 2005)

AMPTE/CCE-CHEM Measurements  
Averaged over 2 years  
(De Michelis, Daglis, Consolini, JGR, 1999)
Disturbed Storm Time Index ($D_{st}$):

$$\Delta B_H = \left(\frac{\mu_0}{2}\right) \times \frac{I_{RC}}{R_{rc}}$$

measured near equator plus Earth’s induction fields!
(LDX: $\Delta I_F \approx -0.25 I_{rc}$)

Dessler-Parker-Sckopke:

Energy = $0.54 \text{ GJ/A} \times I_{RC}$
(LDX: $0.12 \text{ J/A}$)

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Centrally-Peaked Proton Pressure (Even with Plasma Sheet, Outer-Edge, Source!)

AMPTE/CCE-CHEM Measurements
“Quiet Conditions” $I_{RC} \sim 1 \text{ MA}$
(De Michelis, Daglis, Consolini, JGR, 1999)

$P \sim L^{-3.3}$

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Where is the Ring Current?

(I. Karim, 2007)

- 8 flux loops
- 9 normal-B sensors
- 9 tangential-B sensors
- Constant flux constraint on superconducting dipole
- Isotropic now ($P_\perp > P_\parallel$ in future)
- 26 measurements;
  3 unknowns: ($p_0, \psi_0, g$) ...

\[
p(\psi) = p_0 \left( \frac{\psi - \psi_{fcoil}}{\psi_0 - \psi_{fcoil}} \right)^\alpha \left( \frac{\psi}{\psi_0} \right)^4 g
\]
“Best Fit” Anisotropic Equilibrium: Supported Dipole

Parameter Fit Value
χ² 14.5942
Ip 3356.57
δIf -738.437
P 2
P(perp)/P(∥) 5
R(peak) 0.716667
γ 2.40741
γ/(5/3) 1.44444
Press(Rpeak) 594.78
J Centroid 1.23389
Moment (A m²) 5251.09
Max Perp β 0.267272
Perp β(Rpeak) 0.115559
Avg Perp β 0.0383653
Plasma Volume 28.7984
Energy (J) 306.234
E/Ip (J/kA) 91.2342

Peak pressure “in-between” 2.45 and 6.4 resonance.

Steep Gradient!
High β!
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Where is the High-β Plasma?

X-Ray
E > 40 keV

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Abel Inversion (Equatorial) Show Profiles Highly Peaked Near 2.45 GHz Resonance

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High $\beta$ Anisotropic Pressure Produced when Dipole is Mechanically Supported

The effect of anisotropic pressure is also evident in these contour pictures. The pressure contours do not coincide with the flux contours, and the pressure becomes more localized to the midplane as it becomes more anisotropic.

Plasmas with the highest values of $I_p$ and $b$ are created by combining both 2.45 and 6.4 GHz heating frequencies. However, the X-ray images do not show a clear pressure peak in this case, and we suspect that the pressure profile may not be as well represented by the model profile defined above. The sum of the mean-square deviations between the best-fit model profile and the magnetic measurements doubles as compared with single-frequency heating, and this may be due to the presence of two pressure peaks, one at each resonance location. If $R_{peak}$ is assumed to be midway between the resonances and $p = 2$, then 5 kW of heating creates a best-fit plasma (shot 50513029) with $I_p = 3.5$ kA, $D_{Id} = 2.8$ kA, $W_p = 330$ J, $g = 2.8$, peak perpendicular pressure of 750 Pa, and maximum local beta of $b = (2b^2 + b_{jj})^{1/2}/3 = 21\%$. If $R_{peak}$ moves outward and closer to the 2.45 GHz resonance by 5 cm, the best-fit gives $b = 23\%$; moving it inward by 4 cm towards the 6.4 GHz resonance results in the best-fit of $b = 18\%$.

Figure 2 shows the contours of the best-fit pressure and current profiles for the highest $\beta$ discharge.

**Fig. 1.** Contours of the reconstructed pressure profiles superimposed onto the X-ray images measured during (top) 2.45 GHz heating and (bottom) 6.4 GHz heating.
Newly Installed Internal Flux Loops Couple Better to Plasma Currents

Flux Loop 10
Flux Loop 11
Flux Loop 12

Existing
w/New Loops

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(2) “Self Calibration”

- With a levitated dipole, flux loops respond to control fields
- Control coil only: mutuals and induced eddy currents
- Dipole vertical displacement: mutuals
Typical Levitation

- External flux loop
- Internal flux loop
- \( \approx 10 \text{kA} \cdot \text{turn} \) variation of levitation control
- \( \approx 4 \text{mm} \) motion of 1.1 MA \cdot \text{turn} dipole

**Question:** What is the plasma contribution to magnetic signals?
Response from Levitation Control Coil

- Response from steady levitation current determines mutual inductance
- Response during constant current-ramp drives a constant eddy current

No plasma; No dipole.

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Induced Eddy Currents

- Constant control current ramp drives a constant eddy current

- We need to find the mutual between eddy current and detector & the wall eddy decay time, $T_W$

- The ratio of eddy current pick-up, $M_E I_E$, to $dI_L/dt$ is equal to $T_W M_E K_{LE}$
Eddy Decay
Time ($\tau_w$)

• Take the numerical derivative of $(M_E I_E)$

• Find the ratio of this derivative to the eddy-drive shown below.

• $\tau_w \approx 71$ msec for Flux Loop #5.

\[
\frac{dM_E I_E}{dt} = \frac{1}{\tau_w} \left[ -(M_E I_E) - (\tau_w K_L M_E) \frac{dI_L}{dt} \right]
\]
Dipole Vertical “Jog”

- Without plasma, program a vertical displacement of dipole.
- After subtracting direct response from control coil, determine the response due to $\delta z$.
- For Flux Loop #5, $0.011 \text{ mV} \cdot \text{s/mm}$
# Measured Coupling Coefficients

<table>
<thead>
<tr>
<th>Flux Num</th>
<th>$M_L$ (µH)</th>
<th>$KLME$ (µH)</th>
<th>$T_W$ (ms)</th>
<th>$G_Z$ (mV·s/mm)</th>
</tr>
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<td>48.4</td>
<td>42.0</td>
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<td>2</td>
<td>42.1</td>
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<td>3</td>
<td>35.4</td>
<td>33.5</td>
<td>26.4</td>
<td>-78.7</td>
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<td>4</td>
<td>12.5</td>
<td>11.9</td>
<td>58.2</td>
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<td>5</td>
<td>9.41</td>
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<td>71.3</td>
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<td>6</td>
<td>11.3</td>
<td>10.8</td>
<td>60.5</td>
<td>-30.2</td>
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<tr>
<td>7</td>
<td>1.64</td>
<td>1.11</td>
<td>148.5</td>
<td>85.9</td>
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<td>8</td>
<td>1.15</td>
<td>1.02</td>
<td>118.8</td>
<td>79.9</td>
</tr>
<tr>
<td>9</td>
<td>0.584</td>
<td>0.406</td>
<td>167.3</td>
<td>56.8</td>
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<tr>
<td>10 (5 turns)</td>
<td>334.5</td>
<td>177.8</td>
<td>14.9</td>
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<tr>
<td>11 (10 turns)</td>
<td>79.5</td>
<td>66.8</td>
<td>14.2</td>
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<tr>
<td>13 (30 turns)</td>
<td>14.4</td>
<td>10.2</td>
<td>25.7</td>
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<td>14 (50 turns)</td>
<td>5.94</td>
<td>5.79</td>
<td>25.1</td>
<td>-11,710</td>
</tr>
</tbody>
</table>
(3) How Much Dipole Current?

- Dipole current must be known for equilibrium reconstruction. We “measure” dipole current using gravitational force balance.
- Measured weight of dipole is 565 kg
- Control current required for levitation is
- Measured dipole position \((z = 0)\) gives dipole current of \(1.116 \text{ MA} \cdot \text{turn}\)
F-Coil Charge

\[ m_{FG} = -\frac{dM_{FL}}{dz} \left( I_F(0) - \frac{M_{FL}I_{lev}}{L_F} \right) I_{lev} \]

1.559 kA w M = 565 kg

1.558 kA w M = 565 kg

1.116 MA \cdot \text{turns Charge}
(4) How Much Plasma Current?

- Compute the direct and induced contributions from control coil and dipole displacement
- Least-squares best fit to find plasma dipole moment and location of diamagnetic current profile
Higher Power ECRH and Levitation Produces Peak $\beta \sim 35\%$

**Abstract**

We report the first production of high $\beta$ plasma confined in a fully levitated laboratory dipole using neutral gas fueling and electron cyclotron resonance heating. The pressure results primarily from a population of energetic trapped electrons that is sustained for many seconds of microwave heating provided sufficient neutral gas is supplied to the plasma. As compared to previous studies in which the internal coil was supported, levitation results in improved particle confinement that allows higher-density, high-$\beta$ discharges to be maintained at significantly reduced gas fueling. Elimination of parallel losses coupled with reduced gas leads to improved energy confinement and a dramatic change in the density profile. Improved particle confinement assures stability of the hot electron component at reduced pressure. By eliminating supports used in previous studies, cross-field transport becomes the main loss channel for both the hot and the background species. Interchange stationary density profiles, corresponding to an equal number of particles per flux tube, are commonly observed in levitated plasmas.

**1. Introduction**

The dipole confinement concept [1, 2] was motivated by spacecraft observations of planetary magnetospheres that show centrally-peaked plasma pressure profiles forming naturally when the solar wind drives plasma circulation and heating. Unlike most other approaches to magnetic confinement in which stability requires average good curvature and magnetic shear, MHD stability in a dipole derives from plasma compressibility [3–5]. At marginal stability $(pV)^{\gamma/3} = 0$ (with $p$ the plasma pressure, $V = dl/B$ is the differential flux tube volume, and $\gamma = 5/3$), and an adiabatic exchange of flux tubes does not modify the pressure profile nor degrade energy confinement. Non-linear studies indicate that large-scale convective cells will form when the MHD stability limit is weakly violated, which results in the circulation of plasma between the hot core and the cooler edge region [6]. Studies have also predicted that the confined plasma can be stable to low frequency (drift wave) modes when $\delta = d\ln T_e/d \ln n_e > 2/3$ [7]. The marginally stable case to both drift waves and MHD modes, is thus where:

$$p \propto V^{\gamma}$$ and $$n \propto V^{-1}$$.
(4) How Much Plasma Current?

- Control-coil pickup is large for coils located nearby, at top of vessel.
- High-power levitated discharges have large diamagnetic currents...
- \( I_p \approx 9 \, \text{kA}, 3 \times \text{larger than previous}!! \)
- Plasma stored energy more than 1 kJ
(5) Plasma Equilibrium with Levitated Dipole

- First reconstructions with levitated dipole show best fit profiles are isotropic

- Plasma volume is 40% smaller (less stored energy)

- Best fit isotropic profile is broad for full heating power example: 10 GHz + 6.4 GHz + 2.45 GHz
Peak Pressure at Innermost Closed Field Line

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**Example Reconstruction**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Fit Value</th>
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</thead>
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<td>$\chi^2$</td>
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<tr>
<td>$I_p$</td>
<td>-6581.13</td>
</tr>
<tr>
<td>$\Delta I_f$</td>
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<tr>
<td>$\gamma$</td>
<td>1.25</td>
</tr>
<tr>
<td>$\gamma/(5/3)$</td>
<td>0.75</td>
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<tr>
<td>$P(\text{perp})/P(|)$</td>
<td>1</td>
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<tr>
<td>$R(\text{peak})$</td>
<td>0.75</td>
</tr>
<tr>
<td>$J$</td>
<td>1.18396</td>
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<tr>
<td>$\text{Moment (A m}^2\text{)}$</td>
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</tr>
<tr>
<td>$\text{Max Perp } \beta$</td>
<td>0.415474</td>
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<tr>
<td>$\text{Perp } \beta(\text{Rpeak})$</td>
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<tr>
<td>$\text{Avg Perp } \beta$</td>
<td>0.147046</td>
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<tr>
<td>$\text{Plasma Volume}$</td>
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<tr>
<td>$\text{Energy (J)}$</td>
<td>736.583</td>
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<tr>
<td>$E/I_p (\text{J}/\text{kA})$</td>
<td>-111.923</td>
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### Best Fits w/Anisotropy

#### Isotropic!

<table>
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<th>Parameter</th>
<th>Fit Value</th>
<th>Fit Value</th>
<th>Fit Value</th>
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<td>$P_{\text{perp}}/P_{</td>
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<td>}$</td>
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<td>$R_{\text{peak}}$</td>
<td>0.75</td>
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<td>Press($R_{\text{peak}}$)</td>
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</table>

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Saturday, November 15, 2008
Remaining “To Do” List

- Finalize free-boundary equilibrium calculations. (Initial results finished...)
- Compare equilibria during supported and non-supported operation. Plasma pressure appears isotropic during levitation!
- Complete eddy-current structure modeling to improve accuracy
- Incorporate additional external and internal magnetic probes.
Summary

- Magnetic reconstruction of the plasma current during dipole levitation requires subtraction of direct and induced pick-up from control coils and dipole position.

- A “self-calibration” procedure using pre-programmed control currents is used to measure the coupling coefficients.

- Plasma currents are measured to exceed 9 kA, representing stored energy greater than 1 kJ!!